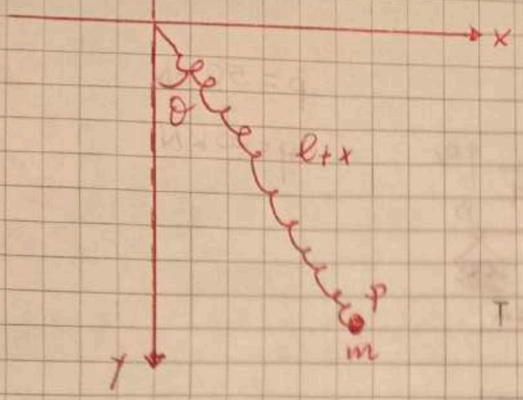
(1)

05/12/2024



$$P\left((l+x)\sin\vartheta\,;\ (l+x)\cos\vartheta\right)$$

$$\vec{v}_P\left(\dot{x}\sin\vartheta+(l+x)\dot{\vartheta}\cos\vartheta\,;\ \dot{x}\cos\vartheta-(l+x)\dot{\vartheta}\sin\vartheta\right)$$

$$T=\frac{1}{2}m v_P^2=\frac{1}{2}m\Big[\dot{x}^2\sin^2\vartheta+(l+x)^2\dot{\vartheta}^2\cos^2\vartheta+$$

$$+2\dot{x}\dot{\vartheta}(l+x)\sin\vartheta\cos\vartheta+\dot{x}^2\cos^2\vartheta+(l+x)^2\dot{\vartheta}^2\sin^2\vartheta+$$

$$-2\dot{x}\dot{\vartheta}(l+x)\sin\vartheta\cos\vartheta\Big]=$$

$$=\frac{1}{2}m\left[\dot{x}^2+(l+x)^2\dot{\vartheta}^2\right]$$

Indicando con U il potenziale totale, si ha:

$$U=U_{el}+U_{grav}=\frac{1}{2}kx^2-mg(l+x)\cos\vartheta$$

Ne viene che la lagrangiana è:

$$\boxed{L=T-U=\frac{1}{2}m\left[\dot{x}^2+(l+x)^2\dot{\vartheta}^2\right]-\frac{1}{2}kx^2+mg(l+x)\cos\vartheta}$$

L'equazione di Lagrange per $q^1=x$ è data da:

$$\left.\begin{aligned}&\frac{\partial L}{\partial\dot{x}}=m\dot{x}\ \Rightarrow\ \frac{d}{dt}\left(\frac{\partial L}{\partial\dot{x}}\right)=m\ddot{x}\\&\frac{\partial L}{\partial x}=m(l+x)\dot{\vartheta}^2-kx+mg\cos\vartheta\end{aligned}\right\}\Rightarrow$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{x}}\right)-\frac{\partial L}{\partial x}=0\ \Rightarrow\ \boxed{m\ddot{x}-m(l+x)\dot{\vartheta}^2+kx-mg\cos\vartheta=0}$$

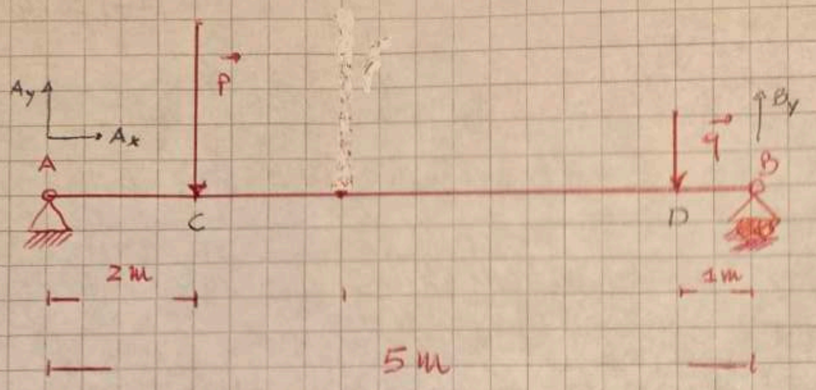
L'equazione di Lagrange per $q^2=\vartheta$ è data da:

$$\left.\begin{aligned}&\frac{\partial L}{\partial\dot{\vartheta}}=m(l+x)^2\dot{\vartheta}\ \Rightarrow\ \frac{d}{dt}\left(\frac{\partial L}{\partial\dot{x}}\right)=2m(l+x)\dot{x}\dot{\vartheta}+m(l+x)^2\ddot{\vartheta}\\&\frac{\partial L}{\partial\vartheta}=-mg(l+x)\sin\vartheta\end{aligned}\right\}\Rightarrow$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\vartheta}}\right)-\frac{\partial L}{\partial\vartheta}=0\ \Rightarrow\ 2m(l+x)\dot{x}\dot{\vartheta}+m(l+x)^2\ddot{\vartheta}+mg(l+x)\sin\vartheta=0\ \Rightarrow$$

$$\boxed{2\dot{x}\dot{\vartheta}+(l+x)\ddot{\vartheta}+g\sin\vartheta=0}$$

2)



$p = 50\,KN$

$q = 20\,KN$

Detti $\underline{R}_A = (A_x, A_y)$ e $\underline{R}_B = (0; B_y)$ le incognite reazioni vincolari, utilizziamo le equazioni cardinali scegliendo A come polo dei momenti:

$$
\begin{array}{ll}
\text{lungo } x: & \left\{ \begin{array}{l} A_x = 0 \\ A_y + B_y - p - q = 0 \\ (C-A) \times \vec{p} + (D-A) \times \vec{q} + (B-A) \times \underline{R}_B = \underline{0} \quad \Rightarrow \end{array} \right. \\
\text{lungo } y: & \\
\text{momento in } A: &
\end{array}
$$

$$(2\underline{i}) \times (-50K\,\underline{j}) + (4\underline{i}) \times (-20K\,\underline{j}) + (5\underline{i}) \times (B_y\,\underline{j}) = \underline{0} \quad \Rightarrow$$

$$-100K - 80K + 5B_y = 0 \quad \Rightarrow \quad B_y = \frac{180}{5}K = 36\,KN$$

$$A_y = -36K + 50 + 20 = 34\,KN$$

Quindi: $\boxed{\underline{R}_A = (0; 34\,KN) \qquad \underline{R}_B = (0; 36\,KN)}$