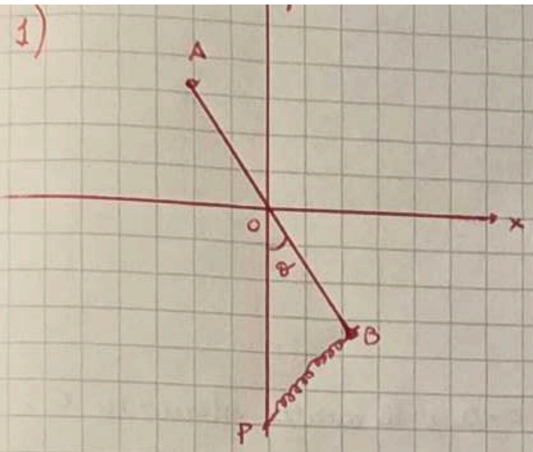
1)

05/06/2026



$\overline{AB} = 2\ell$ $\overline{AO} = \overline{OB}$

$m_A = m$, $m_p = 3m$

$k = \dfrac{mg}{\ell}$

$q_1 = y_P = y$, $q_2 = \vartheta$

$A = (-\ell \sin\vartheta ; \ell\cos\vartheta)$

$B = (\ell \sin\vartheta ; -\ell\cos\vartheta)$ $P = (0; y)$

$\omega = \dot\vartheta \Rightarrow V_A = \omega \ell = \dot\vartheta \ell \Rightarrow V_A^2 = \dot\vartheta^2 \ell^2$

$T_{TOT} = T_A + T_P = \dfrac{1}{2} m \ell^2 \dot\vartheta^2 + \dfrac{3}{2} m \dot y^2$

$|\vec{PB}|^2 = \ell^2 \sin^2\vartheta + y^2 + \ell^2\cos^2\vartheta + 2\ell y\cos\vartheta = y^2 + \ell^2 + 2\ell y\cos\vartheta$

$V_{TOT} = mg\ell\cos\vartheta + 3mgy + \dfrac{mg}{2\ell}\left[y^2 + \ell^2 + 2\ell y\cos\vartheta\right]$

$L : T_{TOT} - V_{TOT} = \dfrac{1}{2} m\ell^2\dot\vartheta^2 + \dfrac{3}{2} m\dot y^2 - mg\ell\cos\vartheta - 3mgy - \dfrac{mg}{2\ell}\left[y^2 + \ell^2 + 2\ell y\cos\vartheta\right]$

Equazione di Lagrange per $q_1 = y$: $\dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot y}\right) - \dfrac{\partial L}{\partial y} = 0 \Rightarrow$

$$\left.\begin{aligned} &\frac{\partial L}{\partial \dot y} = 3m\dot y \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot y}\right) = 3m\ddot y \\ &\frac{\partial L}{\partial y} = -3mg - \frac{mg}{\ell} y - mg\cos\vartheta \end{aligned}\right\} \Rightarrow 3\cancel{m}\ddot y + 3\cancel{m}g + \frac{\cancel{m}g}{\ell} y + \cancel{m}g\cos\vartheta = 0$$

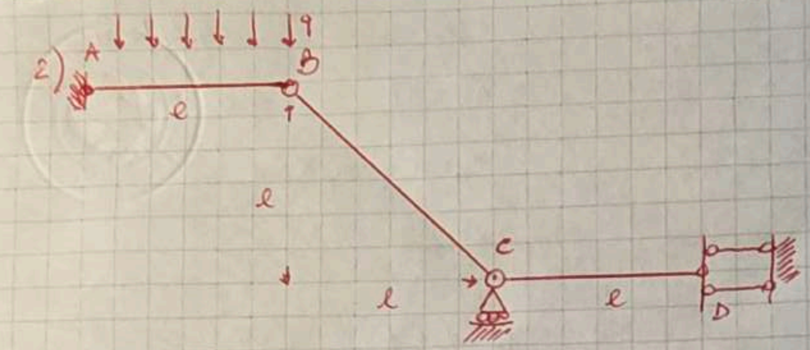
Equazione di Lagrange per $q_2 = \vartheta$: $\dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot\vartheta}\right) - \dfrac{\partial L}{\partial \vartheta} = 0 \Rightarrow$

$$\left.\begin{aligned} &\frac{\partial L}{\partial \dot\vartheta} = m\ell^2\dot\vartheta \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot\vartheta}\right) = m\ell^2\ddot\vartheta \\ &\frac{\partial L}{\partial \vartheta} = mg\ell\sin\vartheta + mgy\sin\vartheta \end{aligned}\right\} \Rightarrow \cancel{m}\ell^2\ddot\vartheta - \cancel{m}g(\ell + y)\sin\vartheta = 0$$

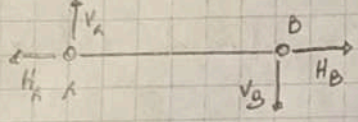
Per le configurazioni di equilibrio si ha:

$$\begin{cases} \dfrac{\partial V}{\partial y} = 3mg + \dfrac{mg}{2\ell}\left[2y + 2\ell\cos\vartheta\right] = 0 \\ \dfrac{\partial V}{\partial \vartheta} = -mg\ell\sin\vartheta - mgy\sin\vartheta = 0 \end{cases} \Rightarrow \begin{cases} y = -\ell(3 + \cos\vartheta) \\ \sin\vartheta = 0 \end{cases} \Rightarrow$$

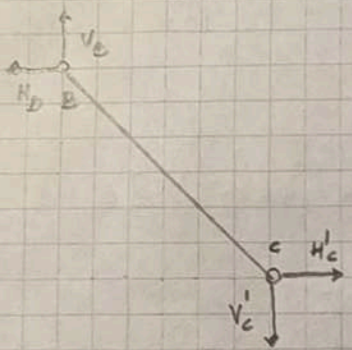
$$\begin{cases} y_1 = -4\ell \\ \vartheta_1 = 0 \end{cases} \vee \begin{cases} y_2 = -2\ell \\ \vartheta_2 = \pi \end{cases}$$



Prendiamo in considerazione le 3 strutture A-B, B-C, C-D e il carrello esterno in C:

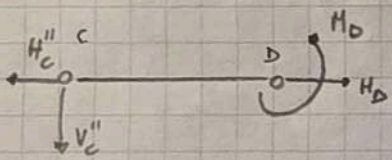


$$\begin{cases} H_B - H_A = 0 \\ V_A - V_B - ql = 0 \\ \text{polo } A: \ (B-A) \times \underline{\Phi}_A + (H-A) \times \underline{q} = 0 \end{cases} \Rightarrow \begin{cases} H_B = H_A \\ V_A - V_B = ql \\ -V_B l - \frac{1}{2} q l^2 = 0 \end{cases}$$

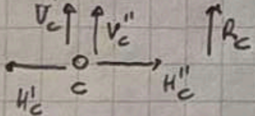


$$\begin{cases} H'_C - H_B = 0 \\ V_B - V'_C = 0 \\ \text{polo } B: \ (C-B) \times \underline{\Phi}'_C = 0 \end{cases} \Rightarrow \begin{cases} H_B = H'_C \\ V_B = V'_C \\ (l\underline{i} - l\underline{j}) \times (H'_C \underline{i} - V'_C \underline{j}) = 0 \end{cases} \Rightarrow$$

$$\begin{cases} H_B = H'_C \\ V_B = V'_C \\ H'_C = V'_C \end{cases}$$



$$\begin{cases} H_D - H''_C = 0 \\ V''_C = 0 \\ M_D = 0 \end{cases}$$



$$\begin{cases} H''_C = H'_C \\ V'_C + V''_C + R_C = 0 \end{cases}$$

Soluzione:

$\underline{\Phi}_A = \left(-\frac{1}{2} ql ; \frac{1}{2} ql\right)$ $\quad \underline{\Phi}_B = \left(-\frac{1}{2} ql ; -\frac{1}{2} ql\right)$ $\quad \underline{\Phi}'_C = \left(-\frac{1}{2} ql ; -\frac{1}{2} ql\right)$

$\underline{\Phi}''_C = \left(-\frac{1}{2} ql ; 0\right)$ $\quad \underline{\Phi}_D = \left(-\frac{1}{2} ql ; 0\right)$ $\quad M_D = 0$

$\underline{R}_C = \left(0 , \frac{1}{2} ql\right)$