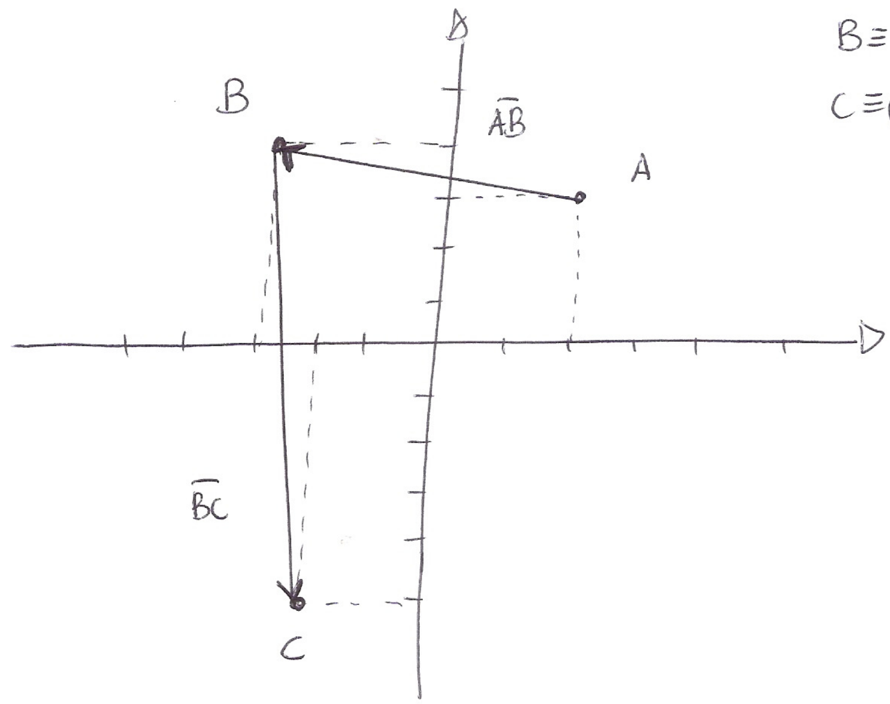
RISOLUZIONE I PROVA DI ESONERO DEL 6-03-2014 FISICA I 12 CFU

ESERCIZIO n. 1

$A \equiv (2, 3)$

$B \equiv (-3, 4)$

$C \equiv (-2, -5)$



$\bar{AB} = (B_x - A_x)\hat{\imath} + (B_y - A_y)\hat{\jmath}$

$\bar{AB} = -5\hat{\imath} + \hat{\jmath}$

RAPP. CARTESIANA

$\bar{BC} = (C_x - B_x)\hat{\imath} + (C_y - B_y)\hat{\jmath}$

$\bar{BC} = +\hat{\imath} - 9\hat{\jmath}$

$|\bar{AB}| = \sqrt{26} = 5.1$

$\theta_{AB} = \operatorname{arctg}\frac{(\bar{AB})_y}{(\bar{AB})_x} \quad \theta_{AB} = 169°$

COMP. POLARI

$|\bar{BC}| = \sqrt{82} = 9.1 \quad \theta_{BC} = 276°$

$$\hat{AB} = \frac{\overline{AB}}{|\overline{AB}|} \qquad \hat{AB} = -1.02\,\hat{i} + 0.2\,\hat{j}$$

VERSORI

$$\hat{BC} = \frac{\overline{BC}}{|\overline{BC}|} \qquad \hat{BC} = +0.11\,\hat{i} - 0.99\,\hat{j}$$

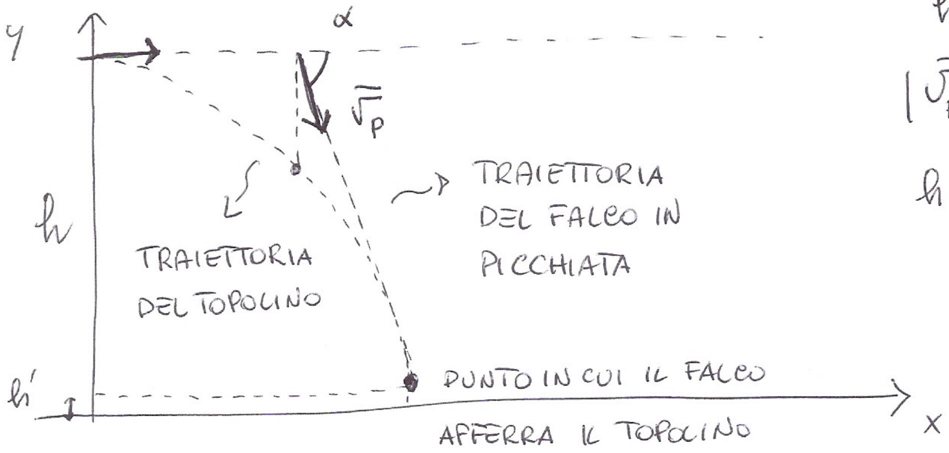
OPERAZIONI

$$\overline{BC} - \overline{AB} = 6\,\hat{i} - 10\,\hat{j}$$

$$\overline{AB} \cdot \overline{BC} = -14$$

$$\overline{BC} \times \overline{AB} = -44\,\hat{K}$$

ESERCIZIO n. 2



$h = 200\ m$

$|\bar{v}_F| = 10\ m/s$

$h' = 3\ m$

SI DETERMININO LE POSIZIONI DEL FALCO E DEL TOPOLINO DOPO $t^* = 2\ s$

MOTO DEL FALCO = MOTO UNIFORME

$$x_F(t) = x_{0F} + v_{Fx}\,t \qquad x_{0F} = 0 \qquad v_{Fx} = +10\ m/s$$

$$x_F(t) = +10t$$

$$x_F(t^*) = +10t^* \qquad x_F(t^*) = 20\ m$$

MOTO DEL TOPOLINO = MOTO UNIF. ACCELERATO

$$\begin{cases} x_T(t) = x_{0T} + v_{0Tx}\,t + \frac{1}{2} a_{Tx} t^2 \\ y_T(t) = y_{0T} + v_{0Ty}\,t + \frac{1}{2} a_{Ty} t^2 \end{cases}$$

$$\begin{cases} x_{0T} = 0 \\ y_{0T} = h \end{cases} \quad \begin{cases} v_{0Tx} = +|\bar{v}_F| = 10\ m/s \\ v_{0Ty} = 0 \end{cases} \quad \begin{cases} a_x = 0 \\ a_y = -g \end{cases}$$

$$\begin{cases} x_T(t) = +10t \\ y_T(t) = 200 - \frac{1}{2}gt^2 \end{cases}$$

$$\begin{cases} x_T(t^*) = 20\ m \\ y_T(t^*) = 180.4\ m \end{cases}$$

VELOCITA' E POSIZIONE DEL TOPOLINO QUANDO IL FALCO INIZIA LA PICCHIATA

$$\begin{cases} v_{xT}(t) = v_{0Tx} \\ v_{yT}(t) = -gt \end{cases}$$

$$\begin{cases} v_{xt}(t^*) = 10\ m/s \\ v_{yt}(t^*) = -19.6\ m/s \end{cases}$$

SI SCRIVANO LE EQUAZIONI DEL MOTO DEL FALCO E DEL TOPOLINO PER $t > t^*$ $(t > 2s)$

FALCO $\Rightarrow$ MOTO UNIFORME CON VELOCITA' $\vec{v}_P$

$$\begin{cases} x_F(t) = x_{0F} + v_{Px}t \\ y_F(t) = y_{0F} + v_{Py}t \end{cases} \qquad \begin{cases} x_{0F} = x_F(t^*) = 20\ m \\ y_{0F} = 200\ m \end{cases}$$

$v_{Px} = |\vec{v}_P| \cos\alpha \qquad v_{Py} = |\vec{v}_P| \sin\alpha$ (INCOGNITE)

TOPOLINO $\Rightarrow$ MOTO UNIFORM. ACCELERATO

$$\begin{cases} x_T(t) = x_{0T} + v_{0xT}t + \frac{1}{2}a_x t^2 \\ y_T(t) = y_{0T} + v_{0yT}t + \frac{1}{2}a_y t^2 \end{cases}$$

$$\begin{cases} x_{0T} = x_T(t^*) = 20\,m \\ y_{0T} = y_T(t^*) = 180.4\,m \end{cases} \qquad \begin{cases} v_{0xT} = v_{xt}(t^*) = 10\,m/s \\ v_{0yT} = v_{yT}(t^*) = -19.6\,m/s \end{cases} \qquad \begin{cases} a_x = 0 \\ a_y = -g \end{cases}$$

$$\begin{cases} x_T(t) = 20 + 10t \\ y_T(t) = 180.4 - 19.6t - \frac{1}{2}gt^2 \end{cases}$$

t^{**} ISTANTE IN CUI IL FALCO AFFERRA IL TOPOLINO

$$x_F(t^{**}) = x_T(t^{**})$$
$$y_F(t^{*}) = y_T(t^{**}) = 3\,m$$

DALLA SECONDA

$$180.4 - 19.6\,t^{*} - \frac{1}{2} g t^{*2} = 3 \qquad t^{**} = 4.3\,s$$

DA CUI

$$x_T(t^{**}) = x_F(t^{**}) = 63\,m$$

SOSTITUENDO NELLE EQUAZIONI DELLA LEGGE DI MOTO DEL FALCO

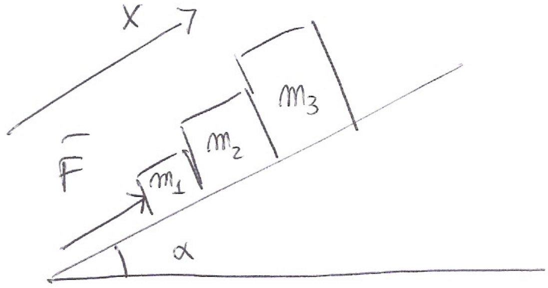
$$20 + v_{Px} t^{**} = 63\,m \qquad v_{Px} = 10\,m/s$$

$$200 + v_{Py} t^{**} = 3\,m \qquad v_{Py} = -45.8\,m/s$$

$$|v_P| = 47\,m/s$$

$$\alpha = \operatorname{arctg} \frac{v_{Px}}{v_{Py}} \qquad \alpha = 78°$$

ESERCIZIO n. 3



$\alpha = 30°$

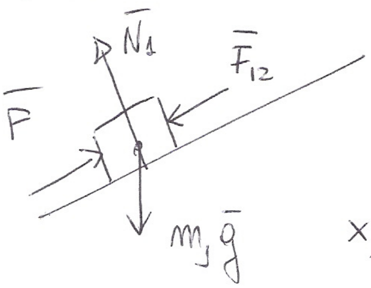
$m_1 = 2\,kg$

$m_2 = 3\,kg$

$m_3 = 4\,kg$

$|\overline{F}| = 40\,N$

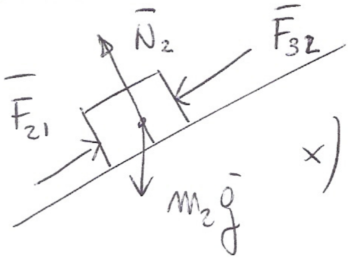
MASSA m_1



$$\overline{F} + \overline{F}_{12} + \overline{N}_1 + m_1\overline{g} = m_1\overline{a}_1$$

$$x)\quad +F - F_{12} - m_1 g \operatorname{sen}\alpha = m_1 a_{1x}$$

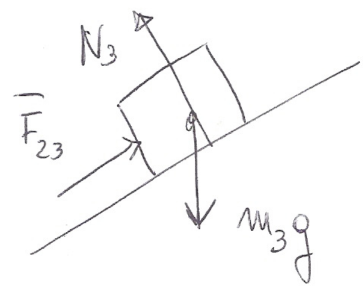
MASSA m_2



$$\overline{F}_{21} + \overline{F}_{32} + m_2\overline{g} = m_2\overline{a}_2$$

$$x)\quad +F_{21} - F_{32} - m_2 g \operatorname{sen}\alpha = m_2 a_{2x}$$

MASSA m_3



$$\overline{F}_{23} + \overline{N}_3 + m_3\overline{g} = m_3\overline{a}_3$$

$$+F_{23} - m_3 g \operatorname{sen}\alpha = m_3 a_{3x}$$

METTENDO A SISTEMA

$$\begin{cases} +F - F_{12} - m_1 g \sin\alpha = m_1 a_{1x} \\ +F_{21} - F_{32} - m_2 g \sin\alpha = m_2 a_{2x} \\ +F_{23} - m_3 g \sin\alpha = m_3 a_{3x} \end{cases}$$

NOTO CHE $|\bar{F}_{12}| = |\bar{F}_{21}|$ $\quad a_{1x} = a_{2x} = a_{3x} = a_x$

$|\bar{F}_{23}| = |\bar{F}_{32}|$

$$\begin{cases} +F - F_{12} - m_1 g \sin\alpha = m_1 a_x \\ +F_{21} - F_{32} - m_2 g \sin\alpha = m_2 a_x \\ +F_{23} - m_3 g \sin\alpha = m_3 a_x \end{cases}$$

$$+F - \cancel{F_{12}} - m_1 g \sin\alpha + \cancel{F_{21}} - \cancel{F_{32}} - m_2 g \sin\alpha + \cancel{F_{23}} - m_3 g \sin\alpha =$$
$$= (m_1 + m_2 + m_3)\, a_x$$

$$F - g \sin\alpha\, (m_1 + m_2 + m_3) = (m_1 + m_2 + m_3)\, a_x$$

$$\frac{F}{m_1 + m_2 + m_3} - g \sin\alpha = a_x \qquad a_x = -0.46\ m/s^2$$

FORZA RISULTANTE SU m_1 $|\bar{F}_{TOT\,1}| = 0.92$
PARALLELA AL PIANO INCLINATO E RIVOLTA VERSO IL BASSO

ANALOGAMENTE PER $\bar{F}_{TOT\,2}$ E $\bar{F}_{TOT\,3}$ CON

$$|\bar{F}_{TOT\,2}| = 1.38\ N \quad e \quad |\bar{F}_{TOT\,3}| = 1.84\ N$$

PER LE FORZE DI CONTATTO DALLA III eq. DEL SISTEMA

$$\bar{F}_{23} = m_3 g \sin\alpha + m_3 a_x$$

$$\bar{F}_{23} = 17.8\ N$$

$$|\bar{F}_{23}| = |\bar{F}_{32}| = 17.8\ N$$

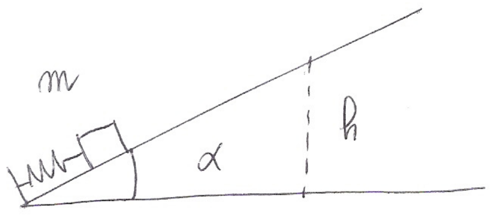
DALLA II eq. DEL SISTEMA

$$\bar{F}_{21} = \bar{F}_{32} + m_2 g \sin\alpha + m_2 a_x$$

$$\bar{F}_{21} = 31.1\ N$$

$$|\bar{F}_{21}| = |\bar{F}_{12}| = 31.1\ N$$

ESERCIZIO n. 4



$\alpha = 60°$

$m = 200\,g$

$K = 1.4\ KN/m$

$d = 10\ cm$

a) IN ASSENZA DI ATTRITO

$$E^i_{TOT} = \frac{1}{2} K d^2 = \frac{1}{2} m v^2$$

$$E^f_{TOT} = m g h_f$$

$$E^i_{TOT} = E^f_{TOT} \qquad \frac{1}{2} K d^2 = m g h_f$$

$$h_f = \frac{K d^2}{2 m g}$$

$$h_f = S_1 \sin\alpha$$

$$S_1 = \frac{h_f}{\sin\alpha}$$

$$S_1 = \frac{K d^2}{2 m g \sin\alpha}$$

CON $K = 1400\ N/m$ $d = 0.1\ m$ $m = 0.2\ Kg$

$$S_1 = 4.16\ m$$

b) IN PRESENZA DI ATTRITO

$$E^i_{TOT} = \frac{1}{2}Kd^2 = \frac{1}{2}mv^2$$

$$E^f_{TOT} = mgh'_f$$

$$E^i_{TOT} = E^f_{TOT} + E_{DISS}$$

$$E_{DISS} = -\mathcal{L}(\bar{f}_{ATT}) \qquad |\bar{f}_{ATT}| \le \mu N$$

$$|\bar{f}_{ATT}| \le \mu mg\cos\alpha$$

$$E_{DISS} = +S_2\,\mu mg\cos\alpha$$

$$E^f_{TOT} = mg\,S_2\sin\alpha + S_2\,\mu mg\cos\alpha$$

$$\frac{1}{2}Kd^2 = mgS_2\sin\alpha + \mu mg\cos\alpha\, S_2$$

$$S_2 = \frac{1}{2}\frac{Kd^2}{mg(\sin\alpha + \mu\cos\alpha)}$$

$$S_2 = 3.35\ m$$