RISOLUZIONE I PROVA DI ESONERO

A.A. 2016-17

ESERCIZIO n. 1

VETTORI IN FORMA CARTES. $\bar{a} = -\hat{\imath} + 2\hat{\jmath} - 4\hat{k}$

$\bar{b} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$

MODULO DI $\bar{a}$ $\quad |\bar{a}| = \sqrt{21}$

MODULO DI $h_1\bar{b}$ $\quad |h_1\bar{b}| = h_1|\bar{b}| = +2\sqrt{14}$

VERSORE DI $\bar{b}$ $\quad \hat{b} = \dfrac{\bar{b}}{|\bar{b}|} \qquad |\bar{b}| = \sqrt{14}$

$$\hat{b} = \frac{3}{\sqrt{14}}\hat{\imath} - \frac{2}{\sqrt{14}}\hat{\jmath} + \frac{1}{\sqrt{14}}\hat{k}$$

VERSORE DI $h_2\bar{a}$ $\quad \widehat{h_2\bar{a}} = h_2\hat{a} = h_2\dfrac{\bar{a}}{|\bar{a}|}$

$h_2 = -2/3$

$|\bar{a}| = \sqrt{21}$

$$\widehat{h_2\bar{a}} = -\frac{2}{3\sqrt{21}}\bar{a}$$

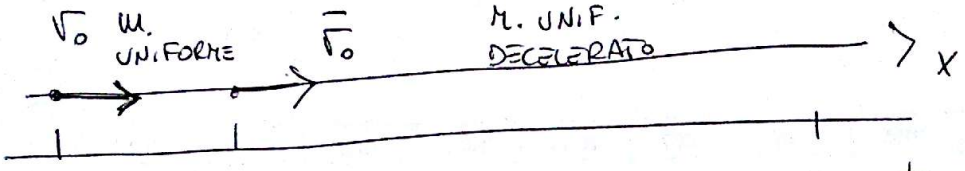
OPERAZIONI

- $h_1\bar{a} - \bar{b} = -5\hat{\imath} + 6\hat{\jmath} - 9\hat{k}$

- $\bar{a} \cdot h_1 \bar{b} = -22$

- $\bar{b} \times h_2 \bar{a} = -4\hat{i} - \frac{22}{3}\hat{j} - \frac{8}{3}\hat{k}$

ESERCIZIO n. 2



$t=0$ t_1 = tempo di reazione 0,7 s t_f istante in cui si ferma

1) Moto uniforme

$x(t) = x_0 + v_{ox} t$ $\quad x_0 = 0$

$v_{ox} = +45$ km/h

$v_{ox} = 12.5$ m/s

$x(t) = 12.5\, t$

$t = t_1 \quad x_0(t_1) = 12.5\,(0.7) = 8.75$ m $= x_1$

2) Moto unif decelerato

$$\begin{cases} x(t) = x_0 + v_{ox} t + \frac{1}{2} a_x t^2 \\ v_x(t) = v_{ox} + a_x t \end{cases}$$

$x_0 = x_1 = 8.75$ m

$v_{ox} = 12.5$ m/s $\Rightarrow$ $\begin{cases} x(t) = 8.75 + 12.5t - \frac{7}{2}t^2 \\ v_x(t) = 12.5 - 7t \end{cases}$

$a_x = -7$ m/s^2

t_f: $v_x(t_f) = 0 \quad \frac{12.5}{7} = t_f \quad t_f = 1.79$ s

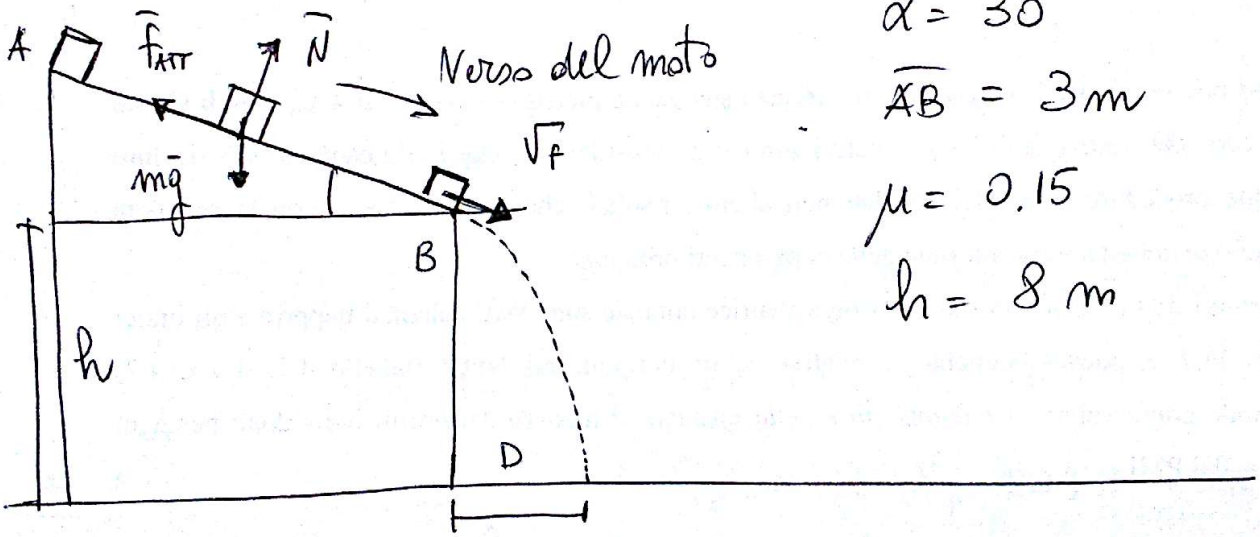
$x_f : \quad x(t_f) = x_f$

$$x_f = 8.75 + 12.5\, t_f - \frac{7}{2} t_f^2$$

$$x_f = 19.9 \text{ m.}$$

La distante percorsa sono 19.9 m

ESERCIZIO n. 3



$\alpha = 30°$

$\overline{AB} = 3m$

$\mu = 0.15$

$h = 8\ m$

$\bar{F} = m\bar{a} \qquad m\bar{g} + \bar{f}_{ATT} + \bar{N} = m\bar{a}$

$$\begin{cases} +mg\sin\alpha - f_{ATT} = ma_x \\ N - mg\cos\alpha = 0 \end{cases}$$

$$|\bar{f}_{ATT}| \le \mu N \qquad |\bar{f}_{ATT}| \le \mu mg\cos\alpha$$

$$+ mg\sin\alpha - \mu mg\cos\alpha = ma_x$$

Lungo il tetto il mattone ha un moto uniformemente accelerato con

$$a = g\sin\alpha - \mu g\cos\alpha = g(\sin\alpha - \mu\cos\alpha)$$

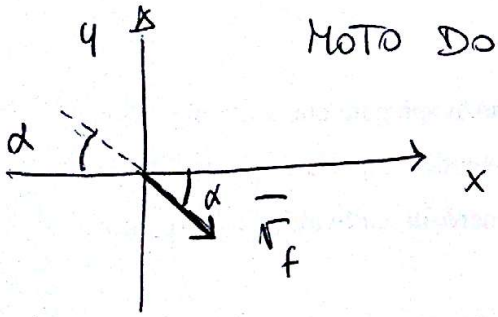
$$a = 3.6\ m/s^2$$

$$\begin{cases} x(t) = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \\ v_x(t) = v_{0x} + a_x t \end{cases} \qquad \begin{aligned} x_0 &= 0 \\ v_{0x} &= 0 \\ a_x &= +3.6\ m/s^2 \end{aligned}$$

t_f : $x(t_f) = 3\ m$

$$x(t_f) = \frac{1}{2}a_x t_f^2 = 3 \qquad t_f^2 = 6/a_x \qquad t_f = 1.3\ s$$

v_f : $v_x(t_f) = a_x t_f \qquad v_x(t_f) = 4.7\ m/s$



MOTO DOPO LA CADUTA DAL TETTO

$$\begin{cases} v_{0x} = +v_f\cos\alpha & v_{0x} = 4.1\ m/s \\ v_{0y} = -v_f\sin\alpha & v_{0y} = -2.35\ m/s \end{cases}$$

$$\begin{cases} x(t) = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \\ y(t) = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 \end{cases} \qquad \begin{cases} v_x(t) = v_{0x} + a_x t \\ v_y(t) = v_{0y} + a_y t \end{cases}$$

$$\begin{cases} x_0 = 0 \\ y_0 = +h \end{cases} \qquad \begin{cases} v_{0x} = +4.1\ m/s \\ v_{0y} = -2.35\ m/s \end{cases} \qquad \begin{cases} a_x = 0 \\ a_y = -g \end{cases}$$

$$\begin{cases} x(t) = +4.1\,t \\ y(t) = +h - 2.35t - \frac{1}{2}gt^2 \end{cases} \qquad \begin{cases} v_x(t) = 4.1 \text{ m/s} \\ v_y(t) = -2.35 - gt \end{cases}$$

TEMPO DI VOLO t_v $\quad t_v : y(t_v) = 0$

$$8 - 2.35\,t_v - \frac{1}{2}9.8\,t_v^2 = 0 \qquad t_v = +1.06$$

DISTANZA DALL'EDIFICIO D $\quad D : x(t_v) = D$

$$x(t_v) = +4.1\,t_v \qquad x(t_v) = 4.35 \text{ m}$$

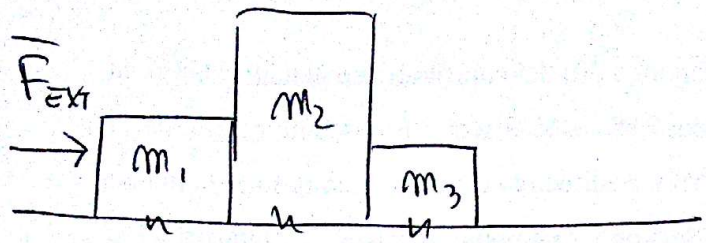
VELOCITÀ FINALE

$$\vec{v}_f : \begin{cases} v_{fx} = 4.1 \text{ m/s} \\ v_{fy} = -2.35 - g\,t_v \end{cases} \qquad \begin{cases} v_{fx} = 4.1 \text{ m/s} \\ v_{fy} = -12.79 \text{ m/s} \end{cases}$$

$$|\vec{v}_f| = 13.4 \text{ m/s}$$

TEMPO TOTALE $\quad t_{TOT} = 1.06 + 1.29 = 2.35 \text{ s}$

Esercizio n. 4



$|\bar{d}| = 30$ cm

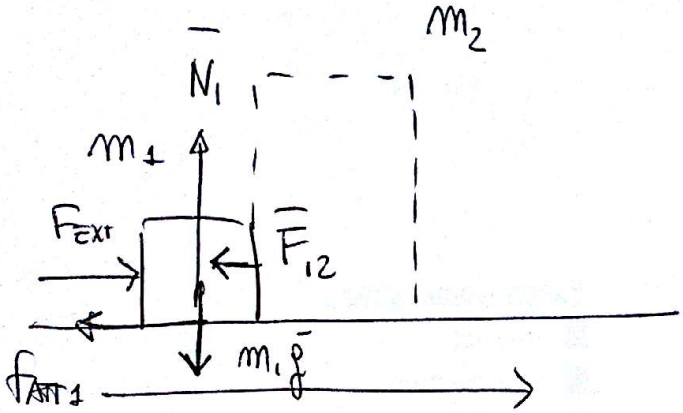
$m_1 = 5$ kg

$m_2 = 10$ kg

$m_3 = 2$ kg

$\mu = 0.2$

$|\bar{F}_{EXT}| = 45$ N



verso del moto

$\bar{F}_{EXT} + \bar{f}_{ATT1} + \bar{N}_1 + m_1\bar{g} + \bar{F}_{12} = m_1\bar{a}$

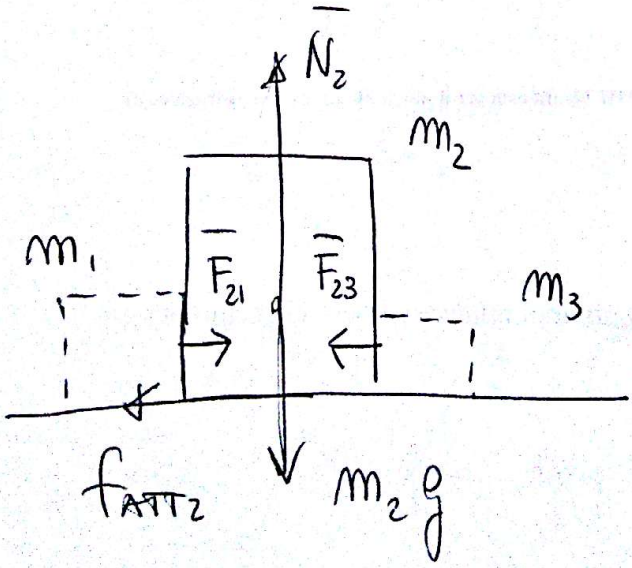
asse x): $+F_{EXT} - f_{ATT1} - F_{12} = m_1 a$

asse y) $N_1 = m_1 g$

$|\bar{f}_{ATT1}| \leq \mu N_1$

$|\bar{f}_{ATT1}| \leq \mu m_1 g$

$$|+F_{EXT} - \mu m_1 g - F_{12} = m_1 a_{1x}|$$



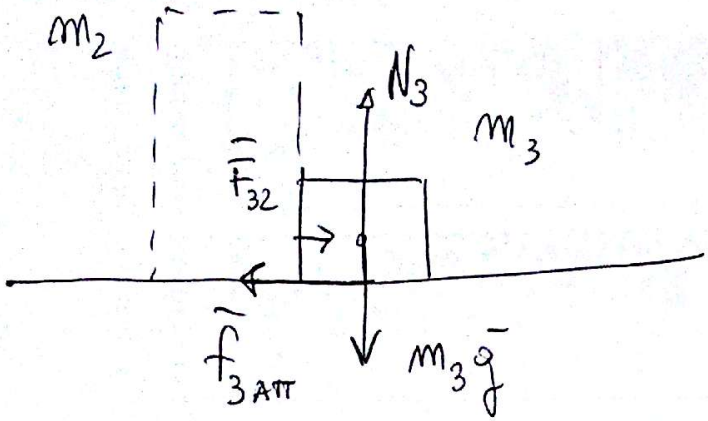
$\bar{F}_{21} + \bar{F}_{23} + \bar{f}_{ATT2} + \bar{N}_2 + m_2\bar{g} = m_2\bar{a}$

$+F_{21} - F_{23} - f_{ATT2} = m_2 a_{2x}$

$|\bar{f}_{ATT2}| \leq \mu N_2 \qquad N_2 = m_2 g$

$$|\bar{f}_{ATT2}| \leq \mu m_2 g$$

$$\boxed{+F_{21} - F_{23} - \mu m_2 g = m_2 a_{2x}}$$



$$\bar{F}_{32} + \bar{N}_3 + m_3\bar{g} + \bar{f}_{ATT3} = m_3\bar{a}$$

$$+F_{32} - f_{ATT3} = m_3 a_{3x}$$

$$|\bar{f}_{ATT3}| \leq \mu N_3 \qquad N_3 = m_3 g$$

$$|f_{ATT3}| \leq \mu m_3 g$$

$$\boxed{F_{32} - \mu m_3 g = m_3 a_{3x}}$$

$|\bar{F}_{12}| = |\bar{F}_{21}| = F_1$ FORZA DI CONTATTO FRA m_1 E m_2

$|\bar{F}_{23}| = |\bar{F}_{32}| = F_2$ FORZA DI CONTATTO FRA m_2 E m_3

$a_{3x} = a_{2x} = a_{1x} = a_x$ LE TRE MASSE HANNO LA STESSA ACCELERAZIONE

$$\begin{cases} +F_{EXT} - \mu m_1 g - F_{12} = m_1 a_x \\ F_1 - F_2 - \mu m_2 g = m_2 a_x \\ F_2 - \mu m_3 g = m_3 a_x \end{cases}$$

SOMMANDO MEMBRO A MEMBRO

$$+F_{EXT} - \mu m_1 g - \cancel{F_1} + \cancel{F_1} - \cancel{F_2} - \mu m_2 g + \cancel{F_2} - \mu m_3 g =$$

$$= m_1 a_x + m_2 a_x + m_3 a_x$$

$$F_{EXT} - \mu g (m_1 + m_2 + m_3) = (m_1 + m_2 + m_3) a_x$$

$$a_1 = \frac{F_{EXT}}{m_1 + m_2 + m_3} - \mu g \qquad a_x = 0.69 \ m/s^2$$

ACCELERAZIONE DI m_2 $\quad a_{2x} = a_x = 0.69 \ m/s^2$

FORZA DI ATTRITO FRA m_3 E IL PIANO $\quad |\bar{f}_{3ATT}| \leq \mu m_3 g$

$$|\bar{f}_{3ATT}| \leq 3.9 \ N$$

FORZA DI CONTATTO FRA m_1 e m_2

$$F_1 = F_{EXT} - \mu m_1 g - m_1 a_x$$

$$F_1 = 31.7 \ N$$

$$\Delta K = \mathcal{L}_{i \to f}$$

$$\mathcal{L}_{i \to f}(\bar{F}_{EXT}) + \mathcal{L}_{i \to f}(\bar{f}_{ATT}) = \mathcal{L}_{i \to f}$$

$$\mathcal{L}_{i \to f} = d\left[F_{EXT} - \mu g (m_1 + m_2 + m_3)\right] \qquad \mathcal{L}_{i \to f} = 2.7 \ J$$

$$\mathcal{L}_{i \to f} > 0 \quad \Delta K > 0 \quad K_f > K_i$$