Risoluzione I prova di esonero

Fisica I 12 CFU a.a. 2017-18

Esercizio n. 1

$\overline{V}_1 = -\hat{i} + 3\hat{j} + 4\hat{k}$ (m/s)

$\overline{V}_2 = 3\hat{i} - 2\hat{j} - 8\hat{k}$ (m^2)

$\overline{V}_3 = 4\hat{i} + 4\hat{j} + 4\hat{k}$ (m/s^2)

Si calcoli $\overline{V}_1 \times (\overline{V}_2 \times \overline{V}_3)$

$$\overline{V}_2 \times \overline{V}_3 \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & -8 \\ 4 & 4 & 4 \end{vmatrix} = \hat{i}(-8+32) - \hat{j}(12+32) + \hat{k}(12$$

$$\overline{V}_2 \times \overline{V}_3 = +24\hat{i} - 44\hat{j} + 20\hat{k} \quad (m^3/s^2)$$

$$\overline{V}_1 \times (\overline{V}_2 \times \overline{V}_3) \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 3 & 4 \\ 24 & -44 & +20 \end{vmatrix} = \hat{i}(60+176) + - \hat{j}(-20-96) + \hat{k}(+44$$

$$\overline{V}_1 \times (\overline{V}_2 \times \overline{V}_3) = 236\hat{i} + 116\hat{j} - 286\hat{k} \quad (m^4/s^3)$$

Si calcoli $(\overline{V}_1 \times \overline{V}_2) \times \overline{V}_3$

$$\overline{V}_1 \times \overline{V}_2 \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & +3 & +4 \\ 3 & -2 & -8 \end{vmatrix} = +\hat{i}(-24+8) - \hat{j}(+8-12) + \hat{k}(2-$$

$$\overline{V}_1 \times \overline{V}_2 = -16\hat{i} + 4\hat{j} - 7\hat{k} \quad (m^3/s$$

$$\left(\bar V_1 \times \bar V_2\right) \times \bar V_3 \Rightarrow \begin{vmatrix} \hat i & \hat j & \hat k \\ -16 & +4 & -7 \\ 4 & 4 & 4 \end{vmatrix} = \hat i\,(16+28) - \hat j\,(-64+28) + \hat k\,(-64-16)$$

$$\left(\bar V_1 \times \bar V_2\right) \times \bar V_3 = +44\,\hat i + 36\,\hat j - 80\,\hat k \quad \left(m^4/s^3\right)$$

PERTANTO $\bar V_1 \times \left(\bar V_2 \times \bar V_3\right) \neq \left(\bar V_1 \times \bar V_2\right) \times \bar V_3$

SI CALCOLI $\bar V_1 \cdot \left(\bar V_2 \times \bar V_3\right)$

$\bar V_1 = -\hat i + 3\,\hat j + 4\,\hat k$

$\bar V_2 \times \bar V_3 = +24\,\hat i - 44\,\hat j + 20\,\hat k$

$\bar V_1 \cdot \left(\bar V_2 \times \bar V_3\right) = -24 - 132 + 80 = -76 \quad m^4/s^3$

SI CALCOLI $\left(\bar V_1 \times \bar V_2\right) \cdot \bar V_3$

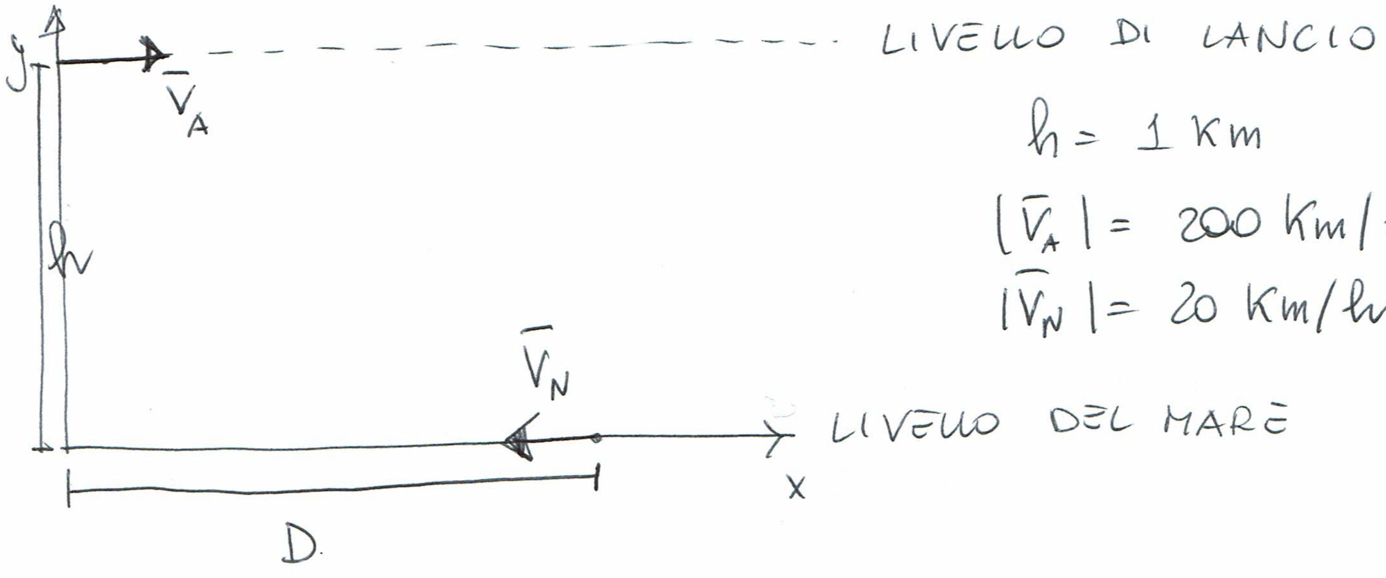
$\bar V_1 \times \bar V_2 = -16\,\hat i + 4\,\hat j - 7\,\hat k$

$\bar V_3 = 4\,\hat i + 4\,\hat j + 4\,\hat k$

$\left(\bar V_1 \times \bar V_2\right) \cdot \bar V_3 = -64 + 16 - 28 = -76 \quad m^4/s^3$

PERTANTO $\bar V_1 \cdot \left(\bar V_2 \times \bar V_3\right) = \left(\bar V_1 \times \bar V_2\right) \cdot \bar V_3$

ESERCIZIO N. 2



$h = 1\ \text{Km}$

$|\bar{V}_A| = 200\ \text{Km/h}$

$|\bar{V}_N| = 20\ \text{Km/h}$

MOTO DEL PACCO → (Moto unif. accelerato)

$$\begin{cases} x_p(t) = x_{op} + v_{opx} t + \frac{1}{2} a_{px} t^2 \\ y_p(t) = y_{op} + v_{opy} t + \frac{1}{2} a_{py} t^2 \end{cases}$$

$$\begin{cases} x_{op} = 0 \\ y_{op} = +h \end{cases} \quad \begin{cases} v_{opx} = +v_A \\ v_{opy} = 0 \end{cases} \quad \begin{cases} a_{px} = 0 \\ a_{py} = -g \end{cases}$$

$$\begin{cases} x_p(t) = +v_A t \\ y_p(t) = +h - \frac{1}{2} g t^2 \end{cases}$$

$$\begin{cases} v_{px}(t) = +v_A \\ v_{py}(t) = -gt \end{cases}$$

MOTO DELLA NAVE → (Moto uniforme)

$x_N(t) = x_{oN} \mp v_N t$

$X_{0N} = D$ (DISTANZA INCOGNITA)

$X_N(t) = D - \bar{v}_N t$

QUANDO IL PACCO CADE SULLA NAVE SIGNIFICA

$$t = t^* \quad \begin{cases} y_P(t^*) = 0 \\ x_P(t^*) = X_N(t^*) \end{cases}$$

$$h - \frac{1}{2} g t^{*} = 0 \implies t^* = \sqrt{\frac{2h}{g}}$$

$$\bar{v}_A t^* = D - v_N t^*$$

$$(\bar{v}_A + v_N)\, t^* = D$$

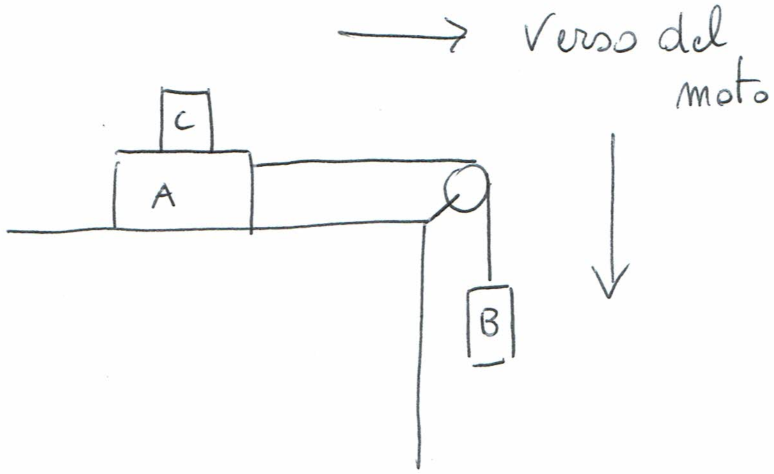
NUMERICAMENTE $\quad h = 1000\ m \quad t^* = 14.3\ s$

$$|\bar{v}_A| = 55.6\ m/s$$

$$|\bar{v}_N| = 5.56\ m/s$$

$$D = 874.6\ m$$

Esercizio n. 3

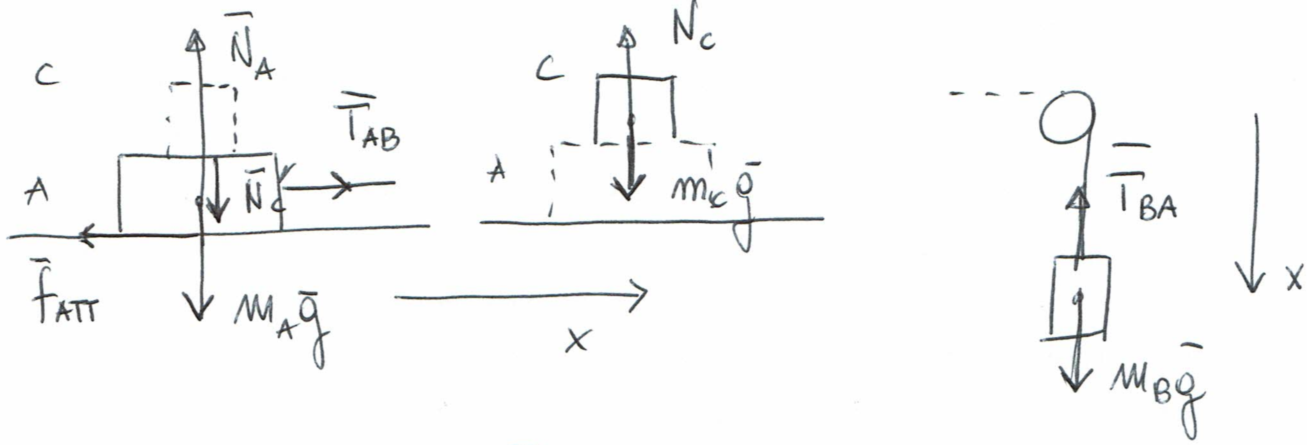


$m_A = 10\ \mathrm{Kg}$

$m_B = 5\ \mathrm{Kg}$

$\mu = 0.20$

DIAGRAMMA DELLE FORZE AGENTI



$$\begin{cases} \bar{f}_{ATT} + \bar{T}_{AB} + m_A\bar{g} + \bar{N}_A + \bar{N}'_C = 0 \\ \bar{N}_C + m_C\bar{g} = 0 \\ \bar{T}_{BA} + m_B\bar{g} = 0 \end{cases}$$

CONDIZIONE DI EQUILIBRIO

SCOMPONENDO PER m_A

$$\begin{cases} -f_{ATT} + T_{AB} = 0 \\ +N_A - m_A g - N'_C = 0 \end{cases} \qquad \begin{cases} T_{AB} = f_{ATT} \\ N_A = m_A g + N'_C \end{cases}$$

$$|\bar{f}_{ATT}| \leq \mu N_A \qquad |\bar{f}_{ATT}| \leq \mu (m_A g + N'_C)$$

$$T_{AB} = \mu(m_A g + N'_c)$$

SCOMPONENDO PER m_C

$$N_c - m_c g = 0 \qquad N_c = m_c g$$

SCOMPONENDO PER m_B

$$-T_{BA} + m_B g = 0 \qquad T_{BA} = m_B g$$

METTENDO INSIEME

$$\begin{cases} T_{AB} = \mu m_A g + \mu N'_c \\ N_c = m_c g \\ T_{BA} = m_B g \end{cases}$$

CONSIDERANDO

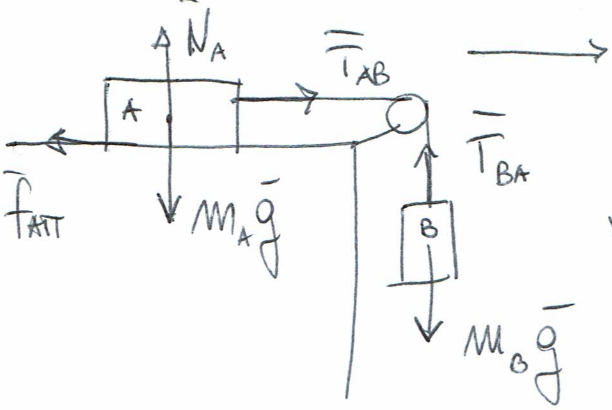
$$|\bar{T}_{AB}| = |\bar{T}_{BA}|$$

$$|\bar{N}_c| = |\bar{N}'_c|$$

$$m_B \cancel{g} = \mu m_A \cancel{g} + \mu m_c \cancel{g}$$

$$\boxed{m_c = \frac{m_B}{\mu} - m_A} \qquad m_c = 15\ Kg$$

SOLLEVATA LA MASSA M_c SI CALCOLA ACCELERAZ
E TENSIONE



verso del moto

$$\bar{f}_{ATT} + \bar{T}_{AB} + m_A\bar{g} + \bar{N}_A = m_A\bar{a}_A$$ $\boxed{\text{MASSA } m}$

$$\begin{cases} -f_{ATT} + T_{AB} = m_A a_{Ax} \\ +N_A - m_A g = 0 \end{cases}$$

$|\bar{f}_{ATT}| \leq \mu N_A$

$|\bar{f}_{ATT}| \leq \mu m_A g$

$$\boxed{-\mu m_A g + T_{AB} = m_A a_{Ax}}$$

$$\bar{T}_{BA} + m_B\bar{g} = m_B\bar{a}_B$$ $\boxed{\text{MASSA } m_B}$

$$\boxed{-T_{BA} + m_B g = m_B a_{Bx}}$$

CONSIDERANDO $|\bar{T}_{BA}| = |\bar{T}_{AB}| = T$ $\quad a_{Ax} = a_{Bx} = a$

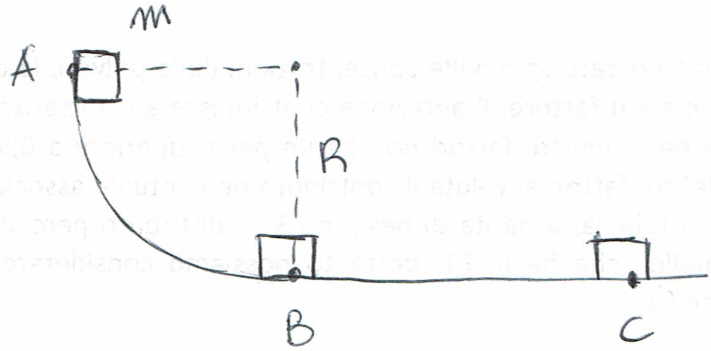
$$\begin{cases} -\mu m_A g + T = m_A a_x \\ -T + m_B g = m_B a_x \end{cases}$$

$$-\mu m_A g + m_B g + \cancel{T} - \cancel{T} = (m_A + m_B)\,a_x$$

$$a_x = \frac{g(m_B - \mu m_A)}{m_A + m_B} \qquad T = m_B(g - a_x)$$

$(a_x = 1.96\ m/s^2\ ;\ T = 39.2\ N$

ESERCIZIO n. 4



$R = 70\ cm$

AB senza attrito

BC con attrito

$\mu_{BC} = 0.25$

m PARTE IN A DA FERMO

NEL TRATTO AB VALE IL PRINCIPIO DI CONSERVAZ. DELL'ENERGIA MECCANICA

$$E^A_{MEC} = E^B_{MEC}$$

$$E^A_{MEC} = K_A + U_A \qquad K_A = 0 \Rightarrow v_A = 0$$

$$U_A = mgh_A \qquad h_A = R$$

$$E^A_{MEC} = mgR$$

$$E^B_{MEC} = K_B + U_B \qquad K_B = \frac{1}{2} m v_B^2$$

$$U_B = 0 \Rightarrow h_B = 0$$

$$E^B_{MEC} = \frac{1}{2} m v_B^2$$

PERTANTO $\quad mgR = \frac{1}{2} m v_B^2$

NEL PUNTO B AVRA' LA VELOCITA' MASSIMA PARI A $v_B = \sqrt{2gR}$ $\quad (v_B = 3.7\ m/s)$

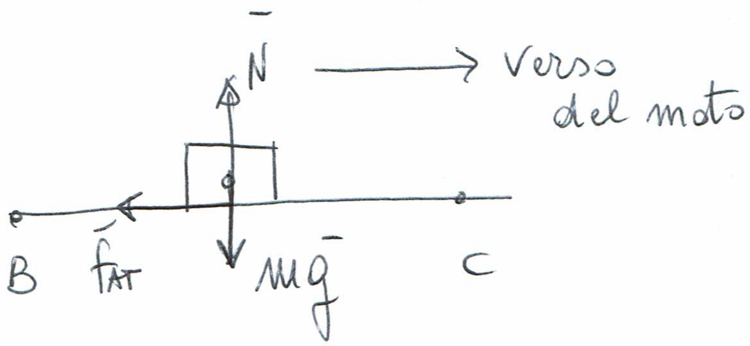
NEL TRATTO BC SI PUO' SCRIVERE UN BILANCIO ENERGETICO

$$E_{MEC}^{B} = E_{MEC}^{C} + E_{DISS\ B \to C}$$

$$E_{MEC}^{B} = \frac{1}{2} m v_B^2$$

$$E_{MEC}^{C} = 0 \Rightarrow v_c = 0$$ SI FERMA IN C

$$E_{DISS\ B \to C} = - \mathcal{L}_{B \to C}(\bar{f}_{ATT})$$



$$|\bar{f}_{ATT}| \le \mu N$$

$$N = mg$$

$$|\bar{f}_{ATT}| \le \mu mg$$

$$\mathcal{L}_{B \to C}(\bar{f}_{ATT}) = + (-\mu mg BC) = -\mu mg BC$$

$$E_{DISS\ B \to C} = \mu mg BC$$

$$\frac{1}{2} m v_B^2 = \mu mg BC$$ CONSIDERANDO $\frac{1}{2} m v_B^2 = mg$

$$\cancel{mg} R = \mu \cancel{mg} BC \qquad BC = R/\mu \qquad BC = 2.8\ m$$

$$AB = \frac{1}{4}(2\pi R) = \frac{\pi R}{2} \qquad AB = 1.1\ m$$

$$S_{TOT} = AB + BC = 3.9\ m$$