RISOLUZIONE PROVA SCRITTA DEL 22-06-2016

ESERCIZIO n. 1

$\bar{A} \equiv (10,\ 307°) \longrightarrow \bar{A} \equiv (+6\ \ -8)$

$\bar{B} = -8\hat{i} + 3\hat{j} \longrightarrow \bar{B} \equiv (-8\ \ +3)$

$\bar{C} \equiv (2,\ -5) \longrightarrow \bar{C} \equiv (+2\ \ -5)$

$\hat{A} = \dfrac{\bar{A}}{|\bar{A}|} \qquad |\bar{A}| = \sqrt{36+64} = \sqrt{100} = 10$

$\hat{A} = \dfrac{6}{10}\hat{i} - \dfrac{8}{10}\hat{j}$

$\hat{B} = \dfrac{\bar{B}}{|\bar{B}|} \qquad |\bar{B}| = \sqrt{64+9} = \sqrt{75} = 8.7$

$\hat{B} = -\dfrac{8}{8.7}\hat{i} + \dfrac{3}{8.7}\hat{j}$

$\hat{C} = \dfrac{\bar{C}}{|\bar{C}|} \qquad |\bar{C}| = \sqrt{4+25} = \sqrt{29} = 5.4$

$\hat{C} = +\dfrac{2}{5.4}\hat{i} - \dfrac{5}{5.4}\hat{j}$

$$\bar{A} - \bar{C} + 2\bar{B} = (+6-2-16)\hat{\imath} + (-8+5+6)\hat{\jmath}$$
$$= -12\hat{\imath} + 3\hat{\jmath}$$

$$\bar{A}\cdot\bar{B} = -48 - 24 = -72$$

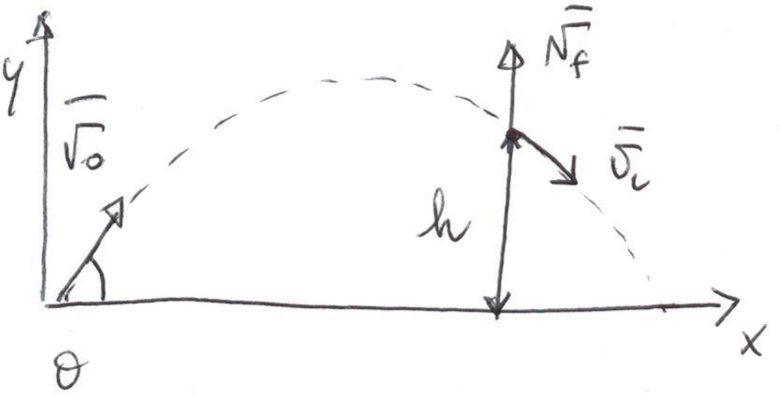
$$\bar{A}\cdot\bar{B}\,(\bar{C}+\bar{A}) = -72(+2+6)\hat{\imath} - 72(-5-8)\hat{\jmath}$$
$$= -72(+8\hat{\imath} - 13\hat{\jmath})$$

$$\bar{B} - \bar{C} = (-8-2)\hat{\imath} + (+3+5)\hat{\jmath} = -10\hat{\imath} + 8\hat{\jmath}$$
$$\bar{A} + \bar{B} = (+6-8)\hat{\imath} + (-8+3)\hat{\jmath} = -2\hat{\imath} - 5\hat{\jmath}$$

$$(\bar{B} - \bar{C}) \times (\bar{A} + \bar{B}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -10 & +8 & 0 \\ -2 & -5 & 0 \end{vmatrix} =$$

$$= \hat{k}(+50 + 16) = +66\hat{k}$$

ESERCIZIO n. 2



$|\bar{v}_0| = 10.6\ m/s$

$\theta = 55°$

$h = 3.05\ m$

$\bar{v}_i$ velocità nell'istante in cui tocca il canestro

$\bar{v}_f$ velocità dopo aver colpito il canestro

$$|\bar{v}_f| = \frac{|\bar{v}_i|}{2}$$

Moto parabolico per trovare $\bar{v}_i$

$$\begin{cases} x(t) = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \\ y(t) = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 \end{cases} \quad \begin{cases} v_x(t) = v_{0x} + a_x t \\ v_y(t) = v_{0y} + a_y t \end{cases}$$

$$\begin{cases} x_0 = 0 \\ y_0 = 0 \end{cases} \quad \begin{cases} v_{0x} = +10.6\cos 55° = 6.1\ m/s \\ v_{0y} = +10.6\sin 55° = 8.7\ m/s \end{cases} \quad \begin{cases} a_x = 0 \\ a_y = -g \end{cases}$$

$$\begin{cases} x(t) = +6.1t \\ y(t) = +8.7t - \frac{1}{2}gt^2 \end{cases} \quad \begin{cases} v_x(t) = 6.1\ m/s \\ v_y(t) = 8.7 - gt \end{cases}$$

Quando tocca il canestro a t^* $y(t^*) = h$

$y(t^*) = 3.05\,m$

$3.05 = 8.7t^* - \frac{1}{2}gt^*$ $\quad t_1 = 0.5\,s$ (fase di salita)

$t_2 = 1.3\,s$ (fase di discesa)

$t^* = t_2 = 1.3\,s$

$v_{ix} = v_x(t^*) = 6.1\ m/s$

$v_{iy} = v_y(t^*) = 8.7 - gt^* = -4.04\ m/s$

$|\vec{v}_i| = \sqrt{(6.1)^2 + (-4.04)^2} = \sqrt{53.5} = 7.3\ m/s$

$|\vec{v}_f| = \frac{|\vec{v}_i|}{2} \quad |\vec{v}_f| = 3.65\ m/s$

Dopo aver toccato il canestro ed essere rimbalzato, si muove di moto rettilineo unif. acc. sotto l'azione di $\vec{g}$ con velocità iniziale pari alla velocità dopo il rimbalzo

$$\begin{cases} y(t) = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 \\ v_y(t) = v_{0y} + a_y t \end{cases} \qquad \begin{cases} y_0 = h \\ v_{0y} = +3.65\ m/s \\ a_y = -g \end{cases}$$

$$\begin{cases} y(t) = h + 3.65t - \frac{1}{2}gt^2 \\ v_y(t) = 3.65 - gt \end{cases}$$

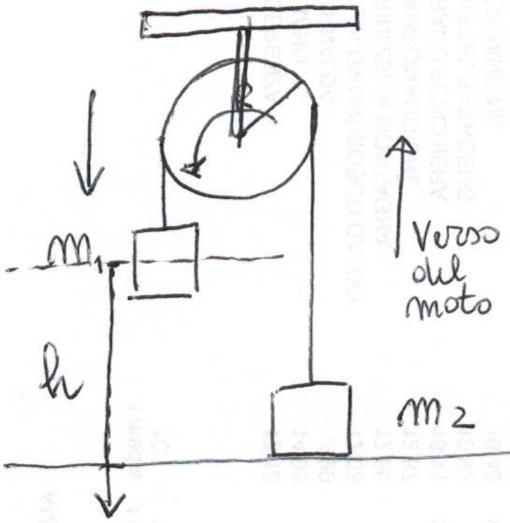
Raggiunge la massima altezza quando la velocità si annulla

$v_y(t^{**}) = 0 \quad 3.65 - g t^{**} \quad t^{**} = 0.37\,s$

$y(t^{**}) = 3.05 + 3.65 t^{**} - \frac{1}{2} g t^{**}$

$y(t^{**}) = 3.72\ m$

ESERCIZIO n. 3



$M = 5\ Kg$

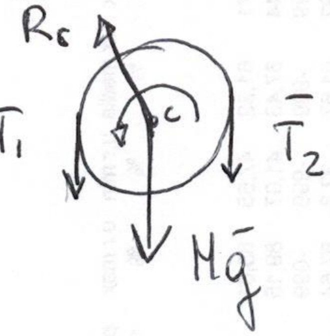
$R = 0.2\ m$

$m_1 = 20\ Kg$

$m_2 = 12.5\ Kg$

$h = 4\ m$

PULEGGIA



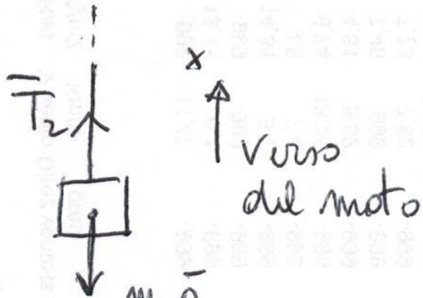
II eq. cardinale - Polo in C

$$+RT_2 + RT_1 = I_c(+\alpha)$$

$$I_c = \frac{1}{2} M R^2$$

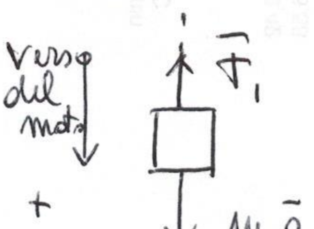
$$\cancel{R}T_2 - \cancel{R}T_1 = \frac{1}{2} M R^{\cancel{2}} \alpha$$

MASSA m_2



$$+T_2 - m_2 g = m_2 a_2$$

MASSA m_1



$$+m_1 g - T_1 = m_1 a_1$$

$$\begin{cases} T_2 - m_2 g = m_2 a_2 \\ m_1 g - T_1 = m_1 a_1 \\ -T_2 + T_1 = \frac{1}{2} M R \alpha \end{cases}$$

$$a_1 = a_2 = R\alpha = a$$

$$\begin{cases} T_2 - m_2 g = m_2 a \\ m_1 g - T_1 = m_1 a \\ -T_2 + T_1 = \frac{1}{2} M a \end{cases}$$

sommando membro a membro

$$\cancel{T_2} - m_2 g + m_1 g - \cancel{T_1} - \cancel{T_2} + \cancel{T_1} = \left(m_1 + m_2 + \frac{M}{2}\right) a$$

$$g(m_1 - m_2) = \left(m_1 + m_2 + M/2\right) a$$

$$a = \frac{(m_1 - m_2)\, g}{m_1 + m_2 + M/2} \qquad a = +2.1\ m/s^2$$

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Moto di m_1 → moto uniform. accelerato

$$y(t) = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$$

$$\begin{cases} y_0 = 0 \\ v_{0y} = 0 \\ a_y = +2.1 \ m/s^2 \end{cases} \qquad y(t) = \frac{1}{2} 2.1 t^2 = 1.05 t^2$$

t^* : $y(t^*) = 4\,m \qquad 4 = 1.05 t^{*2} \qquad t^* = 1.95\,s$

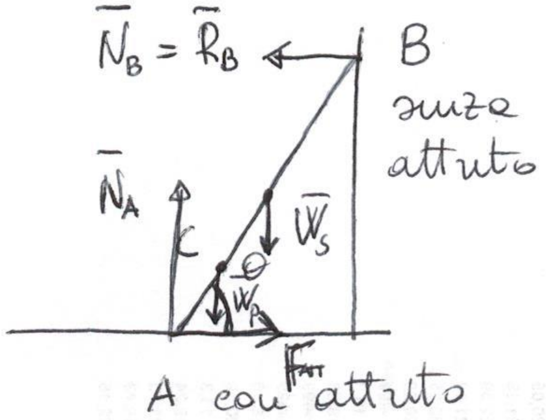
Se la puleggia avesse massa trascurabile

$$a = \frac{m_1 - m_2}{m_1 + m_2} g \qquad a = 2.26 \ m/s^2$$

Quindi $y(t) = \frac{1}{2} 2.26 t^2 = 1.13 t^2$

t^* : $y(t^*) = 4\,m \qquad 4 = 1.13 t^{*2} \qquad t^* = 1.88\,s$

ESERCIZIO n. 4



$\bar{AB} = L = 15\ m$

$W_s = 500\ N$

$\theta = 60°$

$m_p = 81.\ kg$

$AC = 4\ m$

$$\begin{cases} \sum \bar{F}_{EXT} = 0 \\ \sum \bar{m}(\bar{F}_{EXT}) = 0 \end{cases} \qquad \begin{cases} \bar{R}_A = \bar{N}_A + \bar{F}_{ATT} & \text{REAZIONE IN A} \\ \bar{R}_B = \bar{N}_B & \text{REAZIONE IN B} \end{cases}$$

$$\bar{N}_A + \bar{F}_{ATT} + \bar{R}_B + \bar{W}_S + \bar{W}_P = 0$$

asse x $\begin{cases} R_B - F_{ATT} = 0 \\ N_A - W_S - W_P = 0 \end{cases} \quad \begin{cases} R_B = F_{ATT} \\ N_A = W_S + W_P \end{cases}$

$R_B = F_{ATT}$

$N_A = 1294\ N$

Polo in A

$\bar{m}(\bar{N}_A) = 0 \qquad \bar{m}(\bar{W}_P) \to -AC\, W_P \cos\theta$

$\bar{m}(\bar{F}_{ATT}) = 0 \qquad \bar{m}(\bar{W}_S) \to -\frac{L}{2} W_S \cos\theta$

$\bar{m}(\bar{R}_B) \to +L\, R_B \sin\theta$

$$-AC\,W_p \cos\theta - \frac{L}{2} W_s \cos\theta + L R_B \sin\theta = 0$$

$$R_B = \frac{AC\,W_p \cos\theta}{L \sin\theta} + \frac{L W_s \cos\theta}{2L \sin\theta}$$

$R_B = 266\ N$

La forza che il suolo esercita sull'estremo A è $\bar{R}_A = \bar{N}_A + \bar{F}_{ATT}$

$|\bar{R}_A| = \sqrt{N_A^2 + F_{ATT}^2}$ $\quad N_A = 1294\ N$

$F_{ATT} = R_B = 266\ N$

$|\bar{R}_A| = 1321\ N$

Quando $\bar{AC} = 9\ m$ $\quad R_B = 418.25\ N$

Se la scala inizia a scivolare non vale

$|\bar{F}_{ATT}| \leq |\bar{f}_{ATT}|$ $\quad |\bar{f}_{ATT}| \leq \mu_s N_A$

$|\bar{F}_{ATT}| = \mu_s N_A$ $\quad |\bar{F}_{ATT}| = R_B$

$R_B = \mu_s N_A$

$\mu = \frac{R_B}{N_A}$ $\quad \mu_s = 0.32$

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