RISOLUZIONE PROVA SCRITTA DEL 21 GIUGNO 17

ESERCIZIO n. 1

$\bar{a} = (3, -2) \quad \bar{b} = (5, \pi/6) \quad \bar{c} = -\hat{\imath} + 2\hat{\jmath}$

$a_x = +3 \quad b_x = 5 \cos \pi/6 = 4.33 \quad c_x = -1$

$a_y = -2 \quad b_y = 5 \sin \pi/6 = 2.5 \quad c_y = +2$

$|\bar{a}| = \sqrt{9+4} = \sqrt{13}$

$|\bar{b}| = \sqrt{18.7 + 6.25} = \sqrt{25} = 5$

$|\bar{c}| = \sqrt{1+4} = \sqrt{5}$

$\hat{a} = \frac{3}{\sqrt{13}}\hat{\imath} - \frac{2}{\sqrt{13}}\hat{\jmath}$

$\hat{b} = \frac{4.33}{5}\hat{\imath} + \frac{2.5}{5}\hat{\jmath}$

$\hat{c} = -\frac{1}{\sqrt{5}}\hat{\imath} + \frac{2}{\sqrt{5}}\hat{\jmath}$

OPERAZIONI

$\bar{a} + \bar{b} - \bar{c} = (+3 + 4,33 + 1)\hat{\imath} + (-2 + 2.5 + 2)\hat{\jmath}$

$= 8.33\hat{\imath} + 2.5\hat{\jmath}$

$\bar{a} \cdot \bar{c} = (-3) + (-4) = -7$

$(\bar{a} \cdot \bar{c})(-\bar{b}) = 7(4.33\hat{\imath} + 2.5\hat{\jmath})$

$$= 30.3\,\hat{i} + 17.5\,\hat{j}$$

$$\bar{b} \times \bar{c} \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4.33 & 2.5 & 0 \\ -1 & 2 & 0 \end{vmatrix} = \hat{k}\,(8.66 + 2.5) = 11.2\,\hat{k}$$

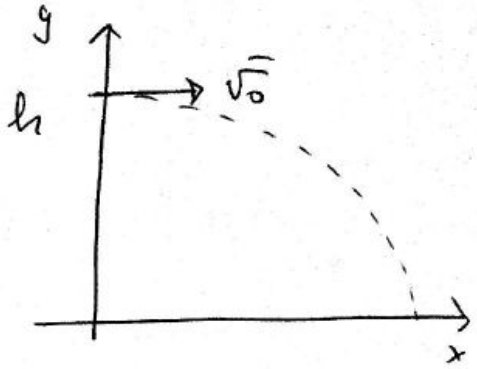
$$(\bar{b} \times \bar{c}) + \bar{a} = +3\,\hat{i} - 2\,\hat{j} + 11.2\,\hat{k}$$

ESERCIZIO n. 2

$m = 0.40\ kg$

$h = 120\ m$

$|\vec{v}_0| = 6\ m/s$ $\quad \vec{v}_0$ ORIZZONTALE



$E^i_{TOT} = K^i + U^i \qquad K_i = \frac{1}{2} m v_0^2 \qquad K_i = 7.2\ J$

$U^i = mgh \qquad U^i = 470.4\ J$

$E^i_{TOT} = 477.6\ J$

$E^f_{TOT} = K^f + U^f \qquad U^f = 0$

$K^f = \frac{1}{2} m v_f^2$

$E^i_{TOT} = E^f_{TOT} \qquad K^f = E^i_{TOT} \qquad K^f = 477.6\ J$

$|\vec{v}_f| = \sqrt{\frac{2K^f}{m}} \qquad |\vec{v}_f| = 48.9\ m/s$

IN UN MOTO PARABOLICO

$\begin{cases} v_x(t) = v_{0x} \text{ COSTANTE} & v_{xf} = |v_f| \cos\vartheta \\ v_y(t) = v_{0y} - gt & \vartheta = 83° \end{cases}$

$$\begin{cases} x(t) = x_0 + v_{0x} t + \frac{1}{2} a_x t^2 \\ y(t) = y_0 + v_{0y} t + \frac{1}{2} a_y t^2 \end{cases}$$

$$\begin{cases} x_0 = 0 \\ v_{0x} = v_0 = 6 \text{ m/s} \\ a_x = 0 \end{cases} \qquad \begin{cases} y_0 = h = 120 \text{ m} \\ v_{0y} = 0 \\ a_y = -g \end{cases}$$

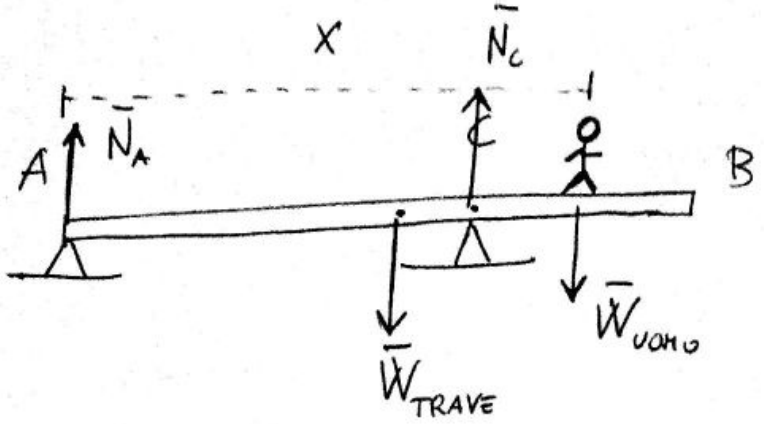
$$\begin{cases} x(t) = 6t \\ y(t) = 120 - \frac{1}{2} g t^2 \end{cases} \qquad t^* \Rightarrow y(t^*) = 0$$

$$t^* = \sqrt{\frac{2 \cdot 120}{g}}$$

$$t^* = 4.95 \text{ s}$$

$$x(t^*) = 6 t^* \qquad x(t^*) = 29.7 \text{ m}$$

ESERCIZIO n. 3



$AB = 4\,m$

$W_{TRAVE} = 1000\,N$

$AC = 2.5\,m$

$W_{UOMO} = 730\,N$

$$\begin{cases} \sum \bar{F}_{EXT} = 0 \\ \sum \bar{m}(\bar{F}_{EXT}) = 0 \end{cases}$$

$$\bar{N}_A + \bar{W}_{TRAVE} + \bar{N}_C + \bar{W}_{UOMO} = 0$$

asse y) $+ N_A - W_{TRAVE} + N_C - W_{UOMO} = 0$

$$N_A = -N_C + W_{TRAVE} + W_{UOMO}$$

Polo in C $\quad \bar{m}(\bar{N}_A) \Rightarrow -AC\,N_A$

$$\bar{m}(\bar{W}_{TRAVE}) \Rightarrow + \left(AC - \frac{AB}{2}\right) W_{TRAVE}$$

$$\bar{m}(\bar{N}_C) \Rightarrow \text{NULLO}$$

$$\bar{m}(W_{UOMO}) \Rightarrow -(X - AC)\,W_{UOMO}$$

$$-AC\,N_A + \left(AC - \frac{AB}{2}\right) W_{TRAVE} - (X - AC)\,W_{UOMO} = 0$$

QUANDO INIZIA A RUOTARE ATTORNO AL PUNTO C N_A SI ANNULLA, QUINDI AFFINCHE' NON RUOTI

$$N_A > 0$$

$$\left(\frac{AC - AB/2}{AC}\right) W_{TRAVE} - \left(\frac{X - AC}{AC}\right) W_{VOLO} > 0$$

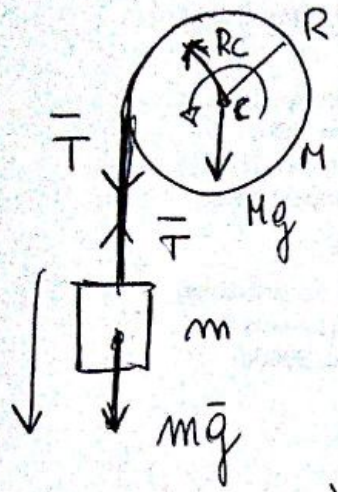
$$X_{MAX} : \quad N_A = 0$$

$$(AC - AB/2)\, W_{TRAVE} - X_{MAX}\, W_{VOLO} + AC\, W_{VOLO} = 0$$

$$X_{MAX} = (AC - AB/2)\frac{W_{TRAVE}}{W_{VOLO}} + AC\frac{\cancel{W_{VOLO}}}{\cancel{W_{VOLO}}}$$

$$X_{MAX} = 3.18\ m$$

ESERCIZIO n. 4



$M = 25\ kg$

$R = 0.50\ m$

$m = 10\ kg$

mano m) $\bar{T} + m\bar{g} = m\bar{a}$

$$\boxed{-T + mg = ma}$$

mano M) $\Sigma \bar{m}(\bar{F}_{ext}) = I\bar{\alpha}$

POLO NEL CENTRO

$\bar{m}(\bar{R}_c) = 0$

$\bar{m}(M\bar{g}) = 0$

$\bar{m}(\bar{T}) \Rightarrow +RT$

$I = \frac{1}{2}MR^2$

$\alpha > 0$

$$\boxed{RT = \frac{1}{2}MR^{\cancel{2}}\alpha}$$

$\alpha = \frac{a}{R}$

$$\begin{cases} -T + mg = ma \\ T = \frac{1}{2}M\cancel{R}\frac{a}{\cancel{R}} \end{cases}$$

$$T = \frac{1}{2} M a$$

$$-\frac{1}{2} M a + mg = ma$$

$$\left(+\frac{M}{2} + m\right) a = mg$$

$$a = \left(\frac{m}{m + M/2}\right) g \qquad a = 4.3 \text{ m/s}^2$$

$$\alpha = \frac{a}{R} \qquad \alpha = 8.7 \text{ rad/s}^2$$

$$T = \frac{1}{2} M a \qquad T = 53.75 \text{ N}$$