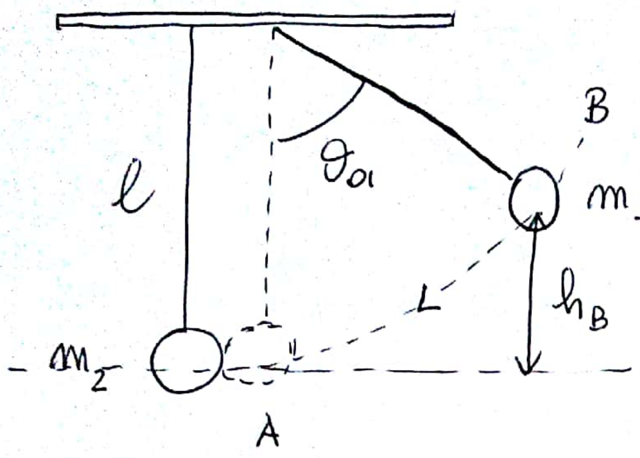
# RISOLUZIONE II PROVA DI VERIFICA DEL 31/05/2018

## ESERCIZIO n. 1




$m_1 = m$

$m_2 = 2m$

$\vartheta_{01} = 45°$ $\quad \vartheta_{02} = 0$

$v_{01} = 0$ $\quad v_{02} = 0$

MASSA $m_1$ DALLA POSIZIONE B ALLA POSIZIONE A

PRINCIPIO DI CONSERV. DELL' ENERGIA MECCANICA

$$m_1 g h_B = \frac{1}{2} m_1 v_{1A}^2$$

$$h_B = \ell - \ell \cos\vartheta_{01} = \ell(1 - \cos\vartheta_{01})$$

$$\cancel{m_1} g \ell (1 - \cos\vartheta_{01}) = \frac{1}{2} \cancel{m_1} v_{1A}^2$$

$$v_{1A} = \sqrt{2g\ell(1-\cos\vartheta_{01})}$$

URTO ELASTICO FRA LE DUE MASSE, SI CONSERVA $\vec{P}^{TOT}$ LUNGO LA DIREZIONE X E, ESSENDO UN URTO ELASTICO SI CONSERVA L'ENERGI CINETICA

$$P^{i}_{TOT\,x} = P^{f}_{TOT\,x}$$

$$m_2 v_{02} + m_1 v_{1A} = m_2 v_1' + m_1 v_1'$$

$$v_{02} = 0 \qquad \boxed{m_1 v_{1A} = m_2 v_1' + m_1 v_1'}$$

$$K^{i}_{TOT} = K^{f}_{TOT}$$

$$\cancel{\frac{1}{2} m_2 v_{02}^2} + \cancel{\frac{1}{2}} m_1 v_{1A}^2 = \cancel{\frac{1}{2}} m_2 v_2'^2 + \cancel{\frac{1}{2}} m_1 v_1'^2$$

$$\boxed{m_1 v_{1A}^2 = m_1 v_1'^2 + m_2 v_2'^2}$$

SI RISOLVE IL SISTEMA RISPETTO ALLE INCOGNITE $v_1'$ e $v_2'$

$$\begin{cases} m_1 v_{1A} - m_1 v_1' = m_2 v_2' \\ m_1 v_{1A}^2 - m_1 v_1'^2 = m_2 v_2'^2 \end{cases}$$

$$\begin{cases} m_1 (v_{1A} - v_1') = m_2 v_2' \\ m_1 (v_{1A}^2 - v_1'^2) = m_2 v_2'^2 \end{cases}$$

$$\begin{cases} m_1 (v_{1A} - v_1') = m_2 v_2' \\ m_1 (v_{1A} - v_1')(v_{1A} + v_1') = m_2 v_2'^2 \end{cases}$$

FACENDO IL RAPPORTO MEMBRO A MEMBRO

$$\frac{\cancel{m_1}\,\cancel{(v_{1A} - v_1')}\,(v_{1A} + v_1')}{\cancel{m_1}\,\cancel{(v_{1A} - v_1')}} = \frac{\cancel{m_2}\,v_2'^{\cancel{2}}}{\cancel{m_2}\,\cancel{v_2'}}$$

IL SISTEMA DA RISOLVERE DIVENTA

$$\begin{cases} m_1 v_{1A} - m_1 v_1' = m_2 v_2' \\ v_{1A} + v_1' = v_2' \end{cases}$$

PER SOSTITUZIONE

$$m_1 v_{1A} - m_1 v_1' = m_2 (v_{1A} + v_1')$$

$$m_1 v_{1A} - m_2 v_{1A} = m_1 v_1' + m_2 v_1'$$

$$(m_1 - m_2) v_{1A} = (m_1 + m_2) v_1'$$

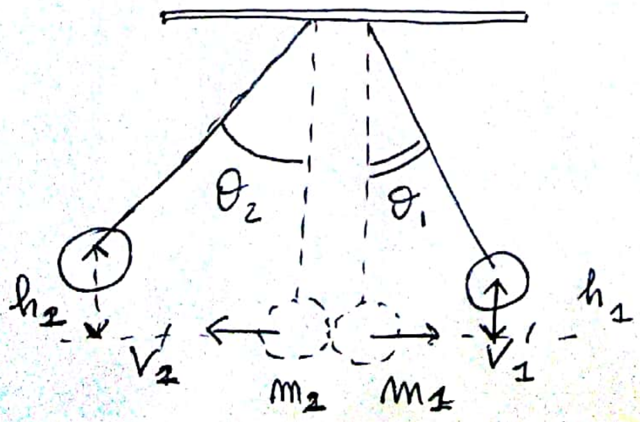
$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1A}$$

CON $m_1 = m$

$m_2 = 2m$

$$v_2' = \left(\frac{2 m_1}{m_1 + m_2}\right) v_{1A}$$

$$\boxed{v_1' = -\frac{1}{3} v_{1A}} \qquad \boxed{v_2' = \frac{2}{3} v_{1A}}$$




DOPO L'URTO PER CIASCUN PENDOLO VALE IL PRINCIPIO DI CONSERV. DELL'ENERGIA MECCANICA

PENDOLO $m_1$ $\qquad \frac{1}{2} \cancel{m_1} v_1'^2 = \cancel{m_1} g h_1$

PENDOLO $m_2$ $\qquad \frac{1}{2} \cancel{m_2} v_2'^2 = \cancel{m_2} g h_2$

$h_1 = \ell(1-\cos\vartheta_1) \qquad v_1'^2 = \frac{1}{9} v_{1A}^2 = \frac{1}{9} 2g\ell(1-\cos\vartheta_{01})$

$h_2 = \ell(1-\cos\vartheta_2) \qquad v_2' = \frac{4}{9} v_{1A}^2 = \frac{4}{9} 2g\ell(1-\cos\vartheta_{01})$

SOSTITUENDO

$$\frac{1}{\cancel{2}}\frac{1}{9}\cancel{2}\cancel{g}\cancel{\ell}(1-\cos\vartheta_{01}) = \cancel{g}\cancel{\ell}(1-\cos\vartheta_1)$$

$$\frac{1}{\cancel{2}}\frac{4}{9}\cancel{2}\cancel{g}\cancel{\ell}(1-\cos\vartheta_{01}) = \cancel{g}\cancel{\ell}(1-\cos\vartheta_2)$$

$$\frac{1}{9}(1-\cos\vartheta_{01}) = 1-\cos\vartheta_1$$

$$\frac{4}{9}(1-\cos\vartheta_{01}) = 1-\cos\vartheta_2$$

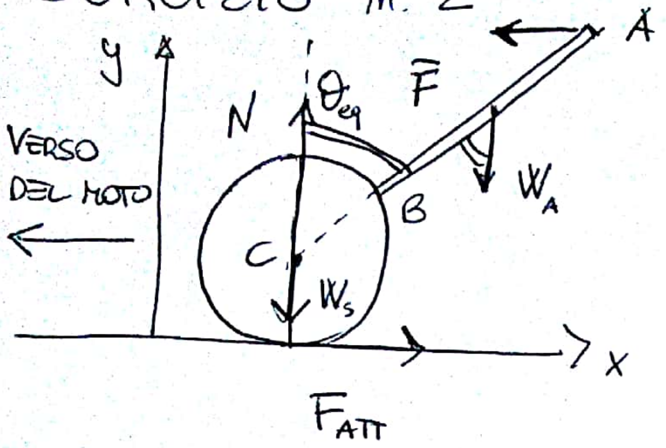
$$\cos\vartheta_1 = 1-\frac{1}{9}+\frac{1}{9}\frac{\sqrt{2}}{2} = \frac{8}{9}+\frac{1}{9}\frac{\sqrt{2}}{2} = 0.968$$

$$\cos\vartheta_2 = 1-\frac{4}{9}+\frac{4}{9}\frac{\sqrt{2}}{2} = \frac{5}{9}+\frac{2}{9}\sqrt{2} = 0.870$$

$\vartheta_1 \simeq 14.5°\qquad h_1 \simeq 0{,}03\,\ell$

$\vartheta_2 \simeq 29.5°\qquad h_2 = 0.13\,\ell$

ESERCIZIO n. 2




$R = 8.5$ cm

$M = 1.4$ Kg

$l = 27.5$ cm

$m = 0.9$ Kg

$\theta_{eq} = 70°$

ALL' EQUILIBRIO $\sum \bar{F}_{EXT} = 0$

$\sum \bar{m}(\bar{F}_{EXT}) = 0$

$$\bar{F} + \bar{F}_{ATT} + \bar{N} + \bar{W}_s + \bar{W}_A = 0$$

$$\begin{cases} + F_{ATT} - F = 0 \\ + N - W_s - W_A = 0 \end{cases} \qquad \begin{cases} F = F_{ATT} \\ N = W_s + W_A \end{cases}$$

POLO NEL CENTRO DELLA SFERA

$\bar{m}(\bar{W}_s) = 0$

$\bar{m}(\bar{N}) = 0$

$\bar{m}(\bar{F}_{ATT}) \Rightarrow + R F_{ATT}$

$\bar{m}(\bar{W}_A) \Rightarrow -\left(R + \frac{AB}{2}\right) W_A \sin\theta_{eq} \qquad \frac{AB}{2} = l$

$-(R + l) W_A \sin\theta_{eq}$

$\bar{m}(\bar{F}) \Rightarrow +(R + AB) F \cos\theta_{eq} \qquad AB = 2l$

$+(R+2\ell)F\cos\vartheta_{eq}$

SOMMANDO

$$+RF_{ATT} - (R+\ell)W_A \sin\vartheta_{eq} + (R+2\ell)F\cos\vartheta_{eq} = 0$$

CON $F_{ATT} = F$

$$+RF + (R+2\ell)F\cos\vartheta_{eq} = (R+\ell)W_A \sin\vartheta_{eq}$$

$$F = \frac{(R+\ell)\sin\vartheta_{eq}}{R+(R+2\ell)\cos\vartheta_{eq}} W_A \qquad F = 9.865\ N$$

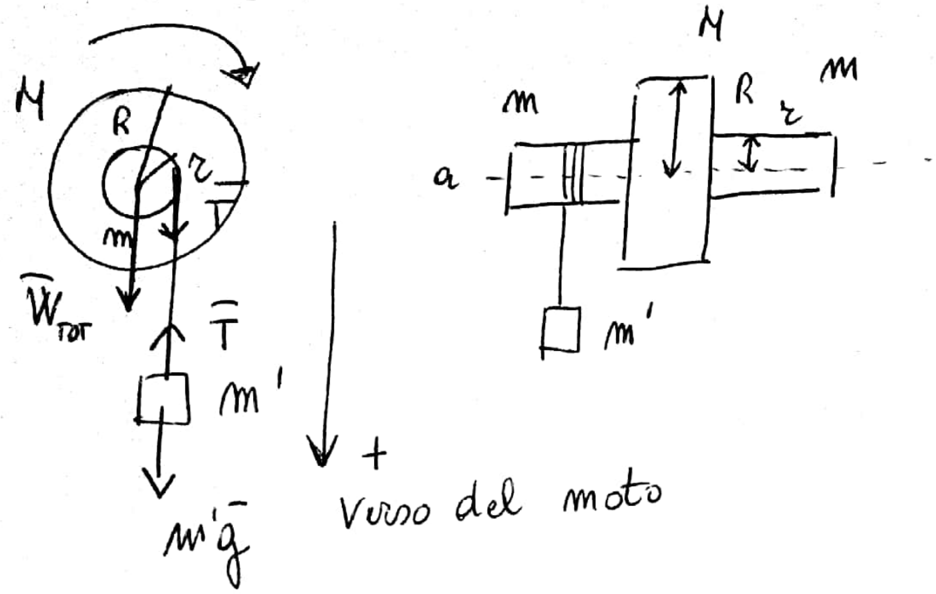
$$F_{ATT} = F \qquad F_{ATT} = 9.865\ N$$

$$N = W_S + W_A \qquad N = 22.54\ N$$

$$|\bar{R}| = \sqrt{F_{ATT}^2 + N^2} \Rightarrow |\bar{R}| = 24.6\ N$$

$$\varphi = \operatorname{arctg}\frac{N}{F_{ATT}} \qquad \varphi \simeq 66°$$

ESERCIZIO n. 3




$M = 1{,}5$ Kg

$m = 0{,}3$ Kg

$m' = 0{,}5$ Kg

$R = 10$ cm

$r = 2{,}5$ cm

PER LA MASSA $m'$ $\quad \bar{T} + m'\bar{g} = m'\bar{a}'$

$$-T + m'g = m'a'$$

PER IL VOLANO $\quad -rT = -I_{TOT}\,\alpha$

CON $\quad I_{TOT} = \frac{1}{2}MR^2 + \frac{1}{2}mr^2 + \frac{1}{2}mr^2$

$$I_{TOT} = \frac{1}{2}MR^2 + mr^2$$

$$rT = \left(\frac{1}{2}MR^2 + mr^2\right)\alpha$$

$a' = \alpha r \qquad T = m'g - m'\alpha r$

$$r(m'g - m'\alpha r) = \left(\frac{1}{2}MR^2 + mr^2\right)\alpha$$

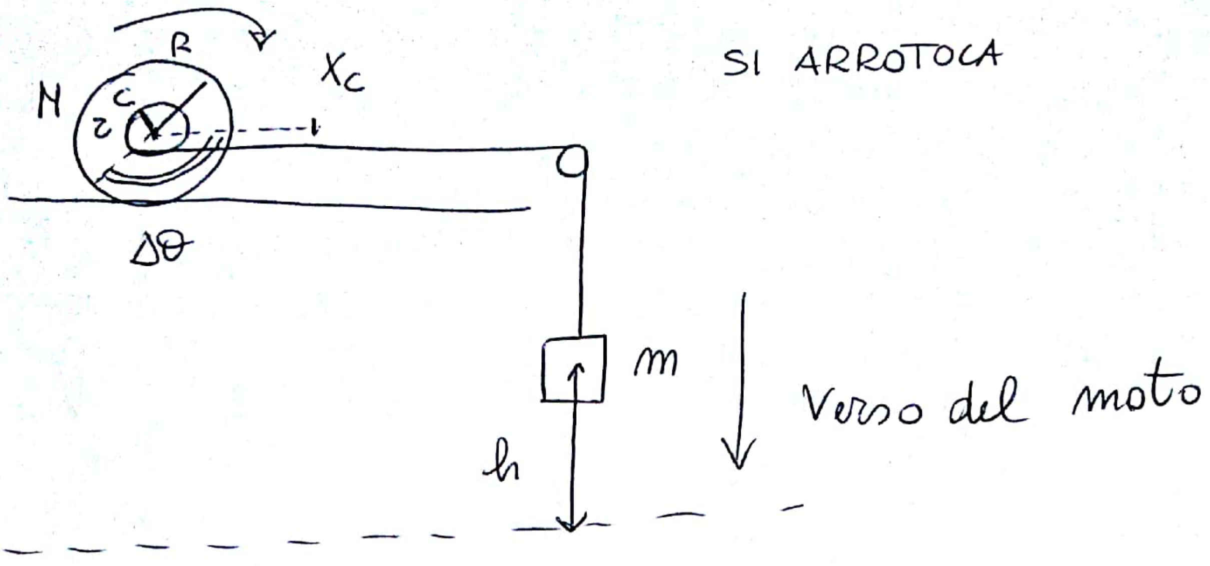
$$rm'g = \left[\frac{1}{2}MR^2 + (m+m')r^2\right]\alpha$$

$$\alpha = \frac{2m'r}{MR^2 + 2(m+m')r^2}\,g \qquad (15.3\ rad/s^2)$$

# ESERCIZIO n. 4

$I_c$ MOMENTO DI INERZIA DEL ROCCHETTO

SI ARROTOLA




$$X_c = R\Delta\theta$$

$h = X_c -$ (IL TRATTO DI CUI SI E' ARROTOLATO $\rightarrow r\Delta\theta$)

$$h = R\Delta\theta - r\Delta\theta = (R-r)\Delta\theta$$

$$\Delta\theta = \frac{X_c}{R}$$

$$h = \left(\frac{R-r}{R}\right)X_c$$

$$v_m = \left(\frac{R-r}{R}\right)v_{cm} \qquad v_{cm} = \omega R$$

APPLICANDO IL PRINCIPIO DI CONSERVAZIONE DELL' ENERGIA

$$\underbrace{mgh}_{\text{EN POTENZIALE DI } m} = \underbrace{\frac{1}{2}m v_m^2}_{\text{ENERGIA CINETICA DI } m} + \underbrace{\frac{1}{2}M v_{cm}^2 + \frac{1}{2}I_c\omega^2}_{\text{ENERGIA CINETICA ROTAZ.+ +TRASLAZ. DEL ROCCHETTO}}$$

SOSTITUENDO $v_{cr}^2 = \frac{R^2}{(R-r)^2} v_m^2$

$$\omega^2 = \frac{v_{cr}^2}{R^2} = \frac{\cancel{R^2}}{\cancel{R^2}(R-r)^2} v_m^2$$

$$mgh = \frac{1}{2} m v_m^2 + \frac{1}{2} M \frac{R^2}{(R-r)^2} v_m^2 + \frac{1}{2} \frac{I_c}{(R-r)^2} v_m^2$$

$$mgh = \frac{1}{2}\left[m + \frac{MR^2}{(R-r)^2} + \frac{I_c}{(R-r)^2}\right] v_m^2$$

$$mgh = \left[\frac{(R-r)^2 m + MR^2 + I_c}{2(R-r)^2}\right] v_m^2$$

$$v_m = \sqrt{\frac{2(R-r)^2 mgh}{(R-r)^2 m + MR^2 + I_c}}$$