RISOLUZIONE PROVA SCRITTA 20/06/2018

ESERCIZIO n. 1

$\bar{a} \equiv (+3 \; -2 \; +4)$ $\quad |\bar{a}| = \sqrt{29}$

$\bar{b} \equiv (0, \; -2 \; +3)$ $\quad |\bar{b}| = \sqrt{13}$

$\bar{c} \equiv (2 \; -3 \; 0)$ $\quad |\bar{c}| = \sqrt{13}$

$\bar{d} \equiv (-3 \; +2 \; -1)$ $\quad |\bar{d}| = \sqrt{14}$

$\hat{a} \equiv \left(+3/\sqrt{29} \; -2/\sqrt{29} \; +4/\sqrt{29}\right)$

$\hat{b} \equiv \left(0, \; -2/\sqrt{13} \; +3/\sqrt{13}\right)$

$\hat{c} \equiv \left(2/\sqrt{13} \; -3/\sqrt{13} \; 0\right)$

$\bar{d} \equiv \left(-3/\sqrt{14} \; +2/\sqrt{14} \; -1/\sqrt{14}\right)$

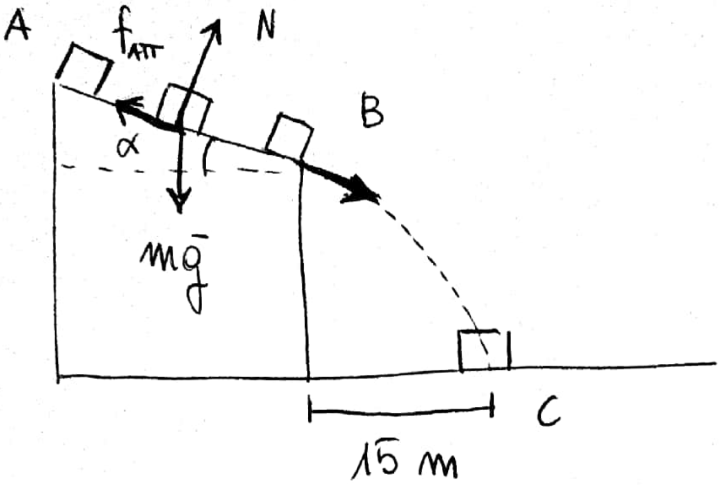
$\bar{a} + \bar{c} - \bar{d} = 8\hat{i} - 7\hat{j} + 5\hat{k}$

$\bar{b} \cdot (\bar{c} - \bar{d}) = 13$

$(\bar{b} \times \bar{c}) + (\bar{d} - \bar{a}) = 3\hat{i} + 10\hat{j} - \hat{k}$

$(\bar{b} + \bar{c}) \cdot (\bar{a} \times \bar{d}) = +33$

ESERCIZIO n. 2



$m = 300\ g$

$\alpha = 30°$

$\mu = 0.15$

$AB = 10\ m$

$\bar{F} = m\bar{a}$ $\quad \bar{N} + \bar{f}_{ATT} + m\bar{g} = m\bar{a}$

$$\begin{array}{l} \text{asse } x) \\ \text{asse } y) \end{array} \begin{cases} -f_{ATT} + mg\sin\alpha = m a_x \\ +N - mg\cos\alpha = 0 \end{cases}$$

$|\bar{f}_{ATT}| \le \mu N \qquad |\bar{f}_{ATT}| \le \mu m g \cos\alpha$

$$-\mu \cancel{m} g\cos\alpha + \cancel{m} g \sin\alpha = \cancel{m} a_x$$

$a_x = g(\sin\alpha - \mu\cos\alpha) \qquad a_x = 3.6\ m/s^2$

Lungo il piano AB:

$$\begin{cases} x(t) = \frac{1}{2} a_x t^2 \\ v_x(t) = a_x t \end{cases} \qquad x(t_D) = L = 10\ m \qquad t_D = \sqrt{2L/a_x} \quad t_D = 2.4\ s$$

$v_F = v_x(t_D)$

$v_F = 8.64\ m/s$

Quando lascia il piano inclinato, il moto avviene sotto l'azione di $\vec{g}$ con una velocità iniziale pari alla velocità in B

$$\begin{cases} v_{0x} = v_F \cos\alpha \\ v_{0y} = -v_F \sin\alpha \end{cases} \qquad \begin{cases} v_{0x} = 7.48 \text{ m/s} \\ v_{0y} = -4.32 \text{ m/s} \end{cases}$$

Pertanto $$\begin{cases} x(t) = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \\ y(t) = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 \end{cases}$$

$$\begin{cases} x_0 = 0 \\ y_0 = h_B \end{cases} \qquad \begin{cases} v_{0x} = 7.48 \text{ m/s} \\ v_{0y} = -4.32 \text{ m/s} \end{cases} \qquad \begin{cases} a_x = 0 \\ a_y = -g \end{cases}$$

$$\begin{cases} x(t) = 7.48\,t \\ y(t) = h_B - 4.32t - \frac{1}{2}9.8t^2 \end{cases}$$

$t = t_v$ tempo di volo $\quad x(t_v) = 15$ m

$t_v = 2$ s

$y(t_v) = 0$

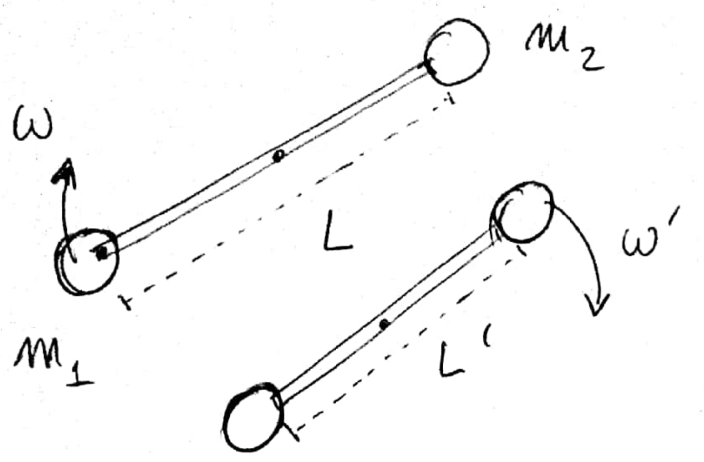
$h_B = 28.2$ m

ENERGIA MECCANICA IN A mgh_A (97.7 J)

ENERGIA MECCANICA IN B $mgh_B + \frac{1}{2}mv_F^2$ (93.9 J)

ENERGIA MECCANICA IN B = ENERGIA MECC. IN C

ESERCIZIO n. 3



$L = 10\ m$

$m_1 = m_2 = m$

$m = 75\ Kg$

$v = 5\ m/s$

$L' = 5\ m$

$$I = m_1 \frac{L^2}{4} + m_2 \frac{L^2}{4} = \cancel{2} m \frac{L^2}{\cancel{4}_2} = m \frac{L^2}{2}$$

$$I = 3750\ Kg\, m^2$$

$$\omega = \frac{|\bar{v}|}{R} \qquad R = L/2 \qquad \omega = 1\ rad/s$$

$$K_{TOT} = \frac{1}{2} I \omega^2 \qquad K_{TOT} = 1875\ J$$

$$|\bar{L}_{TOT}| = I\omega \qquad |\bar{L}_{TOT}| = 3750\ Kg\, m^2\, rad/s$$

SI CONSERVA IL MOMENTO ANGOLARE

$$\bar{L}_{TOT} = \bar{L}'_{TOT} \qquad |\bar{L}_{TOT}| = |\bar{L}'_{TOT}|$$

$$I' = m \frac{L'^2}{2} \qquad L' = 5\ m \qquad I' = 937.5\ Kg\, m^2$$

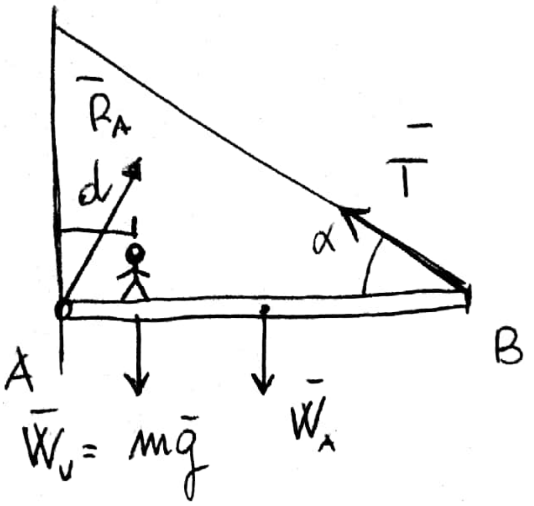
$$\omega' = |\bar{L}'_{TOT}| / I' \qquad \omega' = 4\ rad/s \qquad |\bar{v}'| = \omega' R'$$

$R' = L'/2$

$|\bar{v}'| = 10\ m/s$

$K' = \frac{1}{2} I' \omega'^2 \qquad K' = 7500\ J$

ESERCIZIO n. 4



$AB = L = 8\ m$

$\alpha = 60°$

$d = 2\ m$

$W_A = 200\ N$

$m = 60\ kg$

$T_{MAX} = 750\ N$

(A punto di cerniera)

All'equilibrio $\begin{cases} \sum \bar{F}_{EXT} = 0 \\ \sum \bar{m}(\bar{F}_{EXT}) = 0 \end{cases}$

$\bar{R}_A + \bar{T} + \bar{W}_U + \bar{W}_A = 0 \qquad \begin{cases} R_{Ax} - T_x = 0 \\ R_{Ay} + T_y - W_A - W_U = 0 \end{cases}$

$$\begin{cases} R_{Ax} = T_x \\ R_{Ay} = W_A + W_U - T_y \end{cases}$$

Polo in A $\overline{m}(\overline{R}_A) = 0$

$$\overline{m}(\overline{W}_U) \Rightarrow -d\,W_U$$

$$\overline{m}(\overline{W}_A) \Rightarrow -\frac{L}{2} W_A$$

$$\overline{m}(\overline{T}) \Rightarrow +L\,T \sin\alpha$$

$$L\,T\sin\alpha - d\,W_U - \frac{L}{2} W_A = 0$$

$$T = \frac{d\,W_U + \frac{1}{2} W_A}{L \sin\alpha} \qquad T = 285 \text{ N}$$

$$T_x = T\cos\alpha \qquad T_x = 142.5 \text{ N} \qquad R_x = 142.5 \text{ N}$$

$$T_y = T\sin\alpha \qquad T_y = 247 \text{ N} \qquad R_y = 541 \text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2} \qquad \vartheta = \operatorname{arctg} \frac{R_y}{R_x}$$

$$R = 559 \text{ N} \quad \vartheta = 75°$$

Per $T = T_{MAX}$ $d = d_{MAX}$ massima distanza percorribile sulla trave

$$L\,T_{MAX}\sin\alpha - d_{MAX} W_U - \frac{1}{2} W_A = 0$$

$$d_{MAX} = \frac{L\,T_{MAX}\sin\alpha - \frac{1}{2} W_A}{W_U}$$

$$d_{MAX} = 7.47 \text{ m}$$

0%