

Chapter 3

Properties of Pure Substances

Study Guide in PowerPoint

to accompany

**Thermodynamics: An Engineering Approach, 7th edition
by Yunus A. Çengel and Michael A. Boles**

We now turn our attention to the concept of pure substances and the presentation of their data.

Simple System

A simple system is one in which the effects of motion, viscosity, fluid shear, capillarity, anisotropic stress, and external force fields are absent.

Homogeneous Substance

A substance that has uniform thermodynamic properties throughout is said to be homogeneous.

Pure Substance

A pure substance has a homogeneous and invariable chemical composition and may exist in more than one phase.

Examples:

1. Water (solid, liquid, and vapor phases)
2. Mixture of liquid water and water vapor
3. Carbon dioxide, CO₂
4. Nitrogen, N₂
5. Mixtures of gases, such as air, as long as there is no change of phase.²

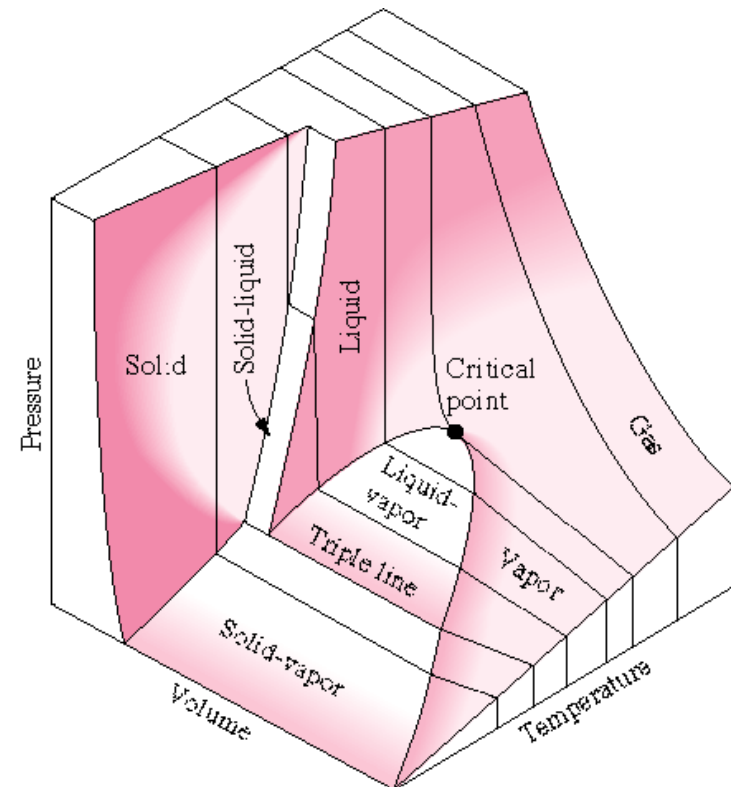
For more information and animations illustrating this topic visit the Animation Library developed by Professor S. Bhattacharjee, San Diego State University, at this link.
test.sdsu.edu/testhome/vtAnimations/index.html

State Postulate

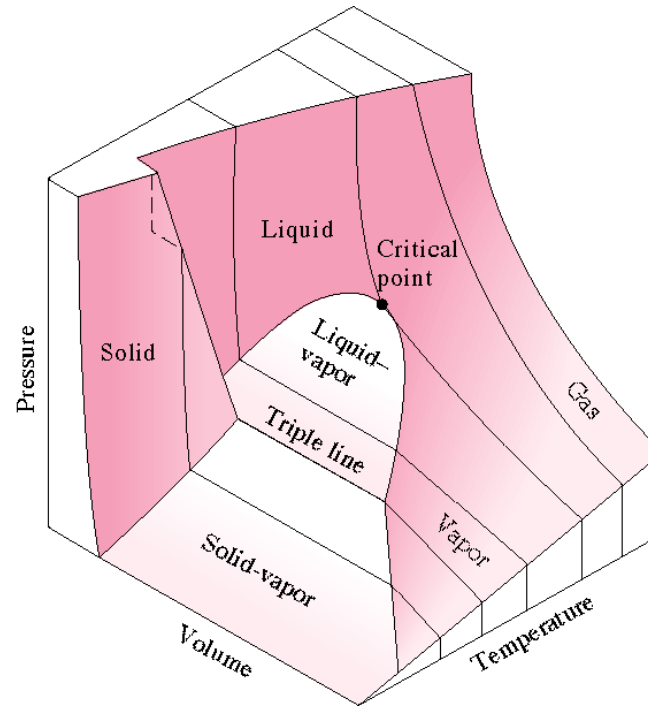
Again, the state postulate for a simple, pure substance states that the equilibrium state can be determined by specifying any two independent intensive properties.

The P - V - T Surface for a Real Substance

◆ P - V - T Surface for a Substance that contracts upon freezing



◆ *P-V-T* Surface for a Substance that expands upon freezing



Real substances that readily change phase from solid to liquid to gas such as water, refrigerant-134a, and ammonia cannot be treated as ideal gases in general. The pressure, volume, temperature relation, or equation of state for these substances is generally very complicated, and the thermodynamic properties are given in table form. The properties of these substances may be illustrated by the functional relation $F(P, v, T)=0$, called an equation of state. The above two figures illustrate the function for a substance that contracts on freezing and a substance that expands on freezing. Constant pressure curves on a temperature-volume diagram are shown in Figure 3-11.

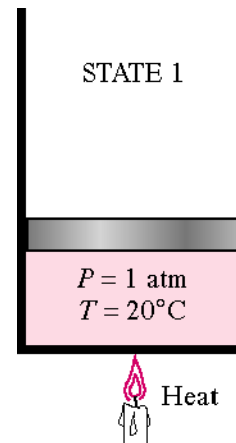
These figures show three regions where a substance like water may exist as a solid, liquid or gas (or vapor). Also these figures show that a substance may exist as a mixture of two phases during phase change, solid-vapor, solid-liquid, and liquid-vapor.

Water may exist in the compressed liquid region, a region where saturated liquid water and saturated water vapor are in equilibrium (called the saturation region), and the superheated vapor region (the solid or ice region is not shown).

Let's consider the results of heating liquid water from 20°C, 1 atm while keeping the pressure constant. We will follow the constant pressure process shown in Figure 3-11. First place liquid water in a piston-cylinder device where a fixed weight is placed on the piston to keep the pressure of the water constant at all times. As liquid water is heated while the pressure is held constant, the following events occur.

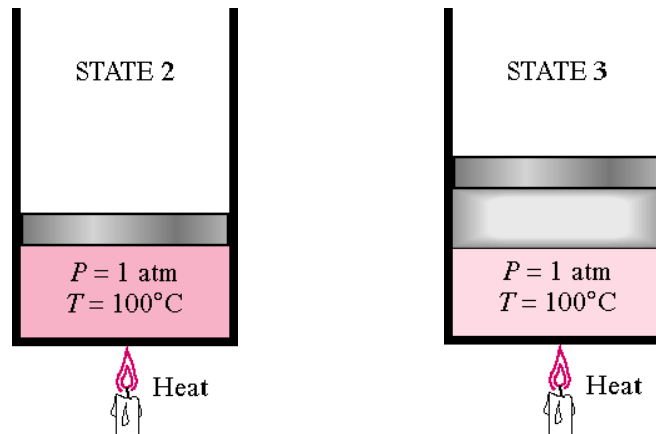
Process 1-2:

The temperature and specific volume will increase from the compressed liquid, or subcooled liquid, state 1, to the saturated liquid state 2. In the compressed liquid region, the properties of the liquid are approximately equal to the properties of the saturated liquid state at the temperature.



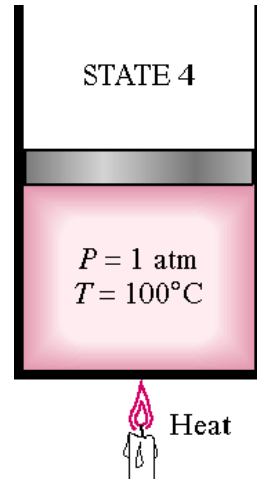
Process 2-3:

At state 2 the liquid has reached the temperature at which it begins to boil, called the saturation temperature, and is said to exist as a saturated liquid. Properties at the saturated liquid state are noted by the subscript **f** and $v_2 = v_f$. During the phase change both the temperature and pressure remain constant (according to the International Temperature Scale of 1990, ITS-90, water boils at $99.975^\circ\text{C} \cong 100^\circ\text{C}$ when the pressure is 1 atm or 101.325 kPa). At state 3 the liquid and vapor phase are in equilibrium and any point on the line between states 2 and 3 has the same temperature and pressure.



Process 3-4:

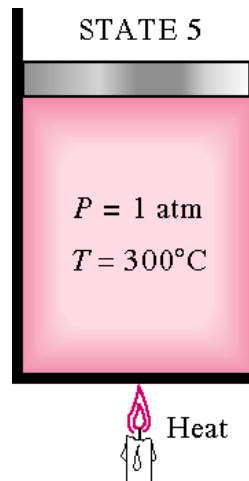
At state 4 a saturated vapor exists and vaporization is complete. The subscript **g** will always denote a saturated vapor state. Note $v_4 = v_g$.



Thermodynamic properties at the saturated liquid state and saturated vapor state are given in Table A-4 as the saturated temperature table and Table A-5 as the saturated pressure table. These tables contain the same information. In Table A-4 the saturation temperature is the independent property, and in Table A-5 the saturation pressure is the independent property. The saturation pressure is the pressure at which phase change will occur for a given temperature. In the saturation region the temperature and pressure are dependent properties; if one is known, then the other is automatically known.

Process 4-5:

If the constant pressure heating is continued, the temperature will begin to increase above the saturation temperature, 100 °C in this example, and the volume also increases. State 5 is called a superheated state because T_5 is greater than the saturation temperature for the pressure and the vapor is not about to condense. Thermodynamic properties for water in the superheated region are found in the superheated steam tables, Table A-6.



This constant pressure heating process is illustrated in the following figure.

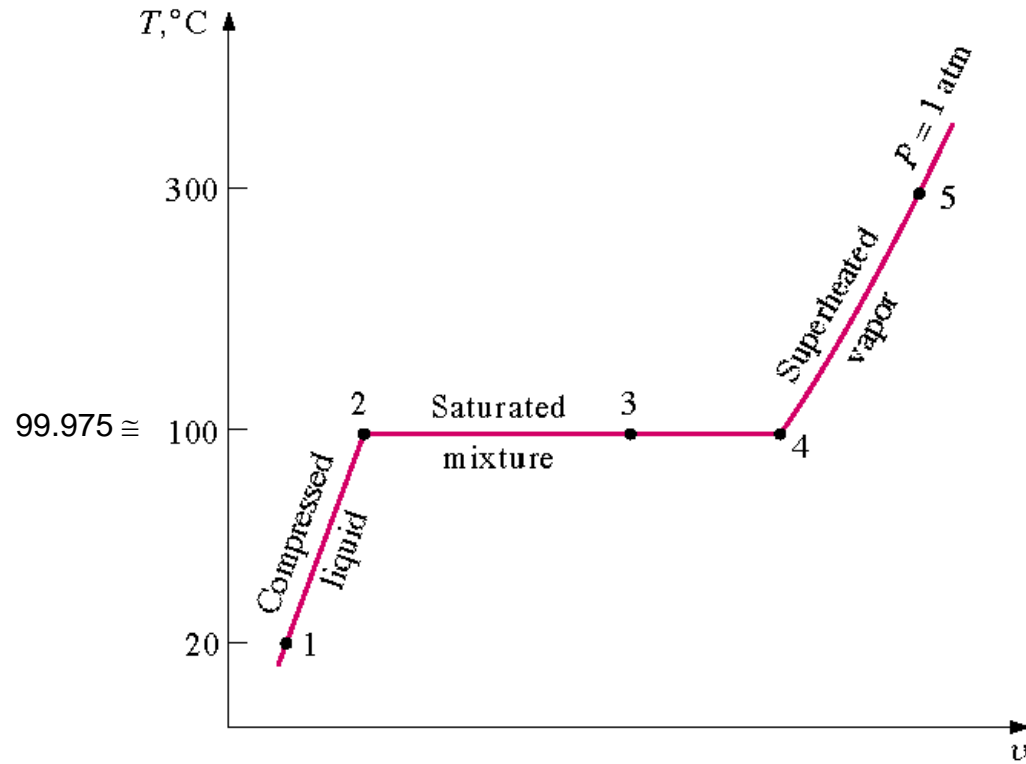
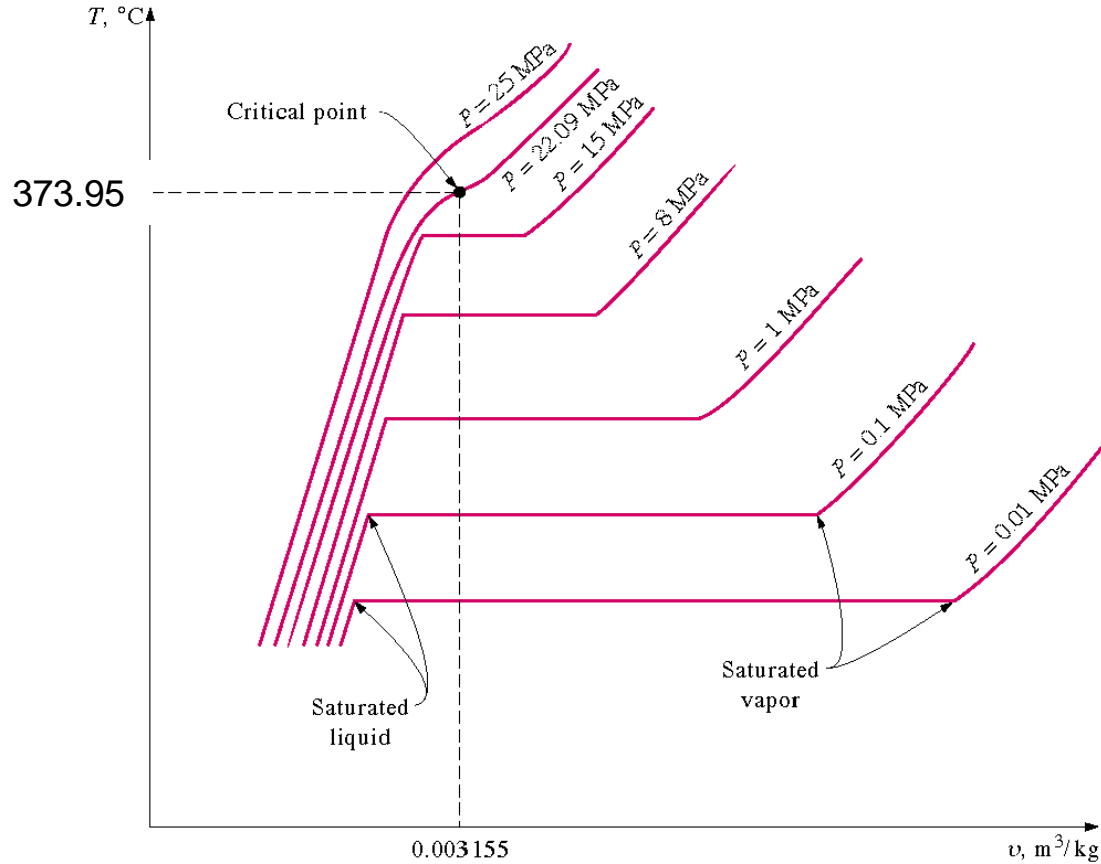
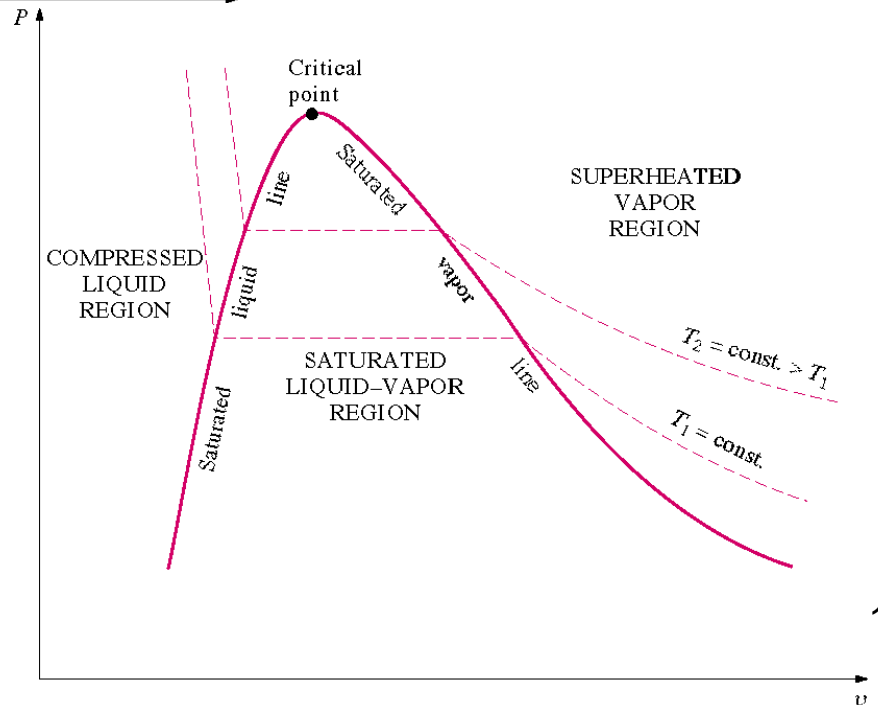
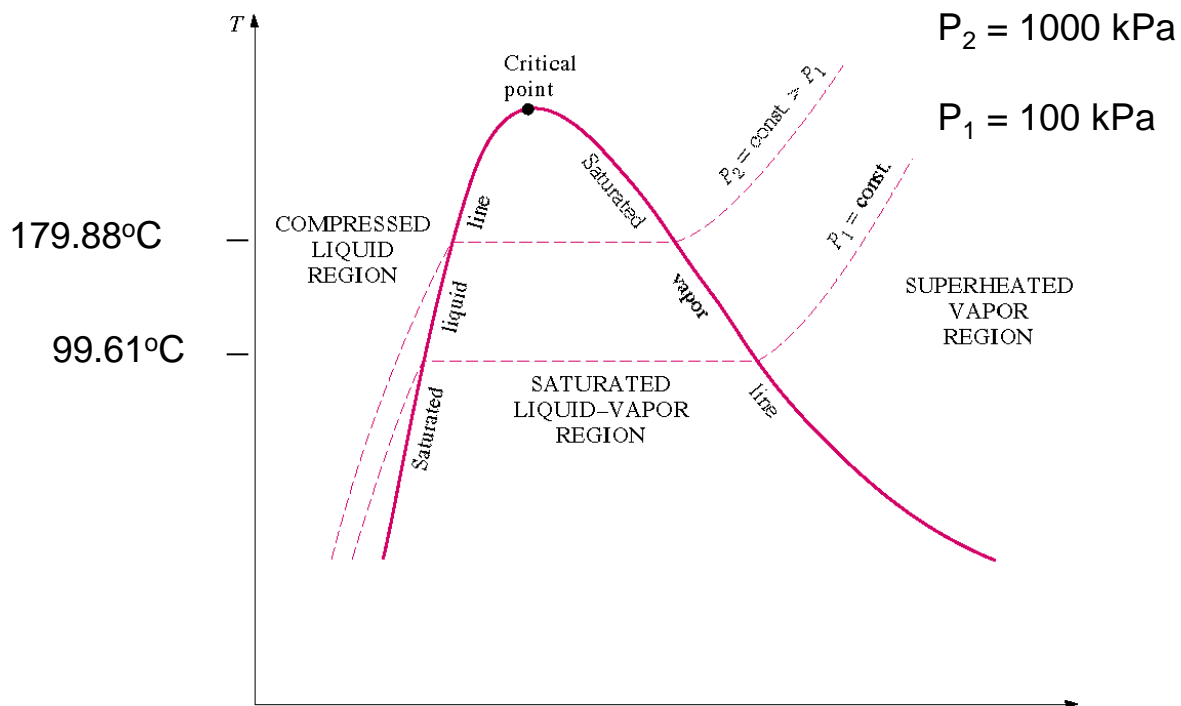


Figure 3-11

Consider repeating this process for other constant pressure lines as shown below.



If all of the saturated liquid states are connected, the saturated liquid line is established. If all of the saturated vapor states are connected, the saturated vapor line is established. These two lines intersect at the critical point and form what is often called the “steam dome.” The region between the saturated liquid line and the saturated vapor line is called by these terms: saturated liquid-vapor mixture region, wet region (i.e., a mixture of saturated liquid and saturated vapor), two-phase region, and just the saturation region. Notice that the trend of the temperature following a constant pressure line is to increase with increasing volume and the trend of the pressure following a constant temperature line is to decrease with increasing volume.



The region to the left of the saturated liquid line and below the critical temperature is called the compressed liquid region. The region to the right of the saturated vapor line and above the critical temperature is called the superheated region. See Table A-1 for the critical point data for selected substances.

Review the P - v diagrams for substances that contract on freezing and those that expand on freezing given in Figure 3-21 and Figure 3-22.

At temperatures and pressures above the critical point, the phase transition from liquid to vapor is no longer discrete.

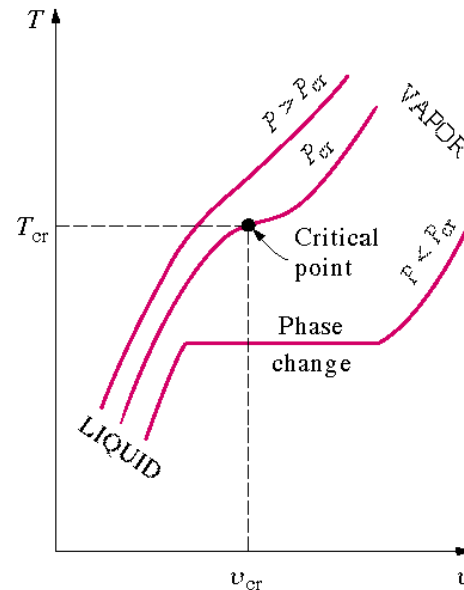
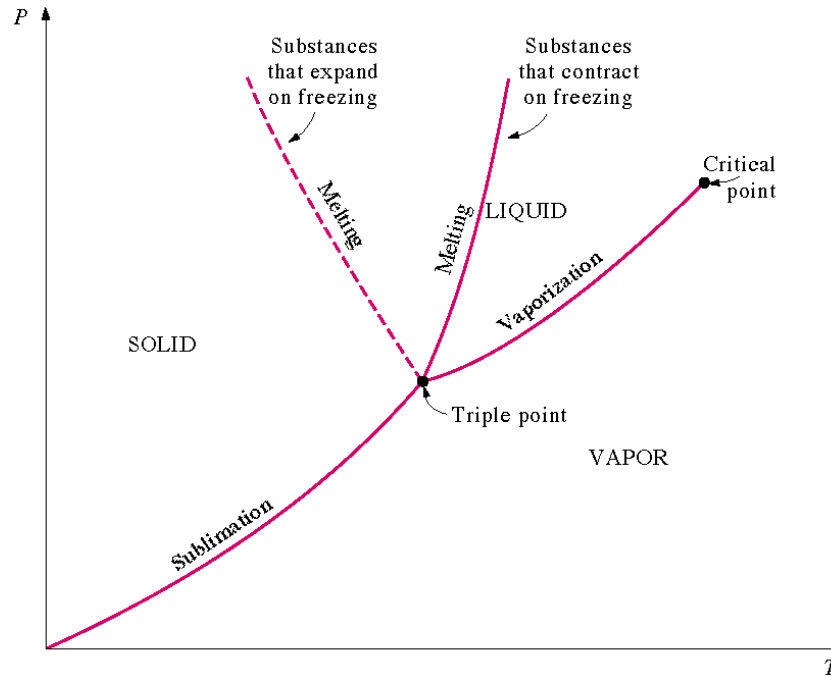


Figure 3-25 shows the P - T diagram, often called the phase diagram, for pure substances that contract and expand upon freezing.

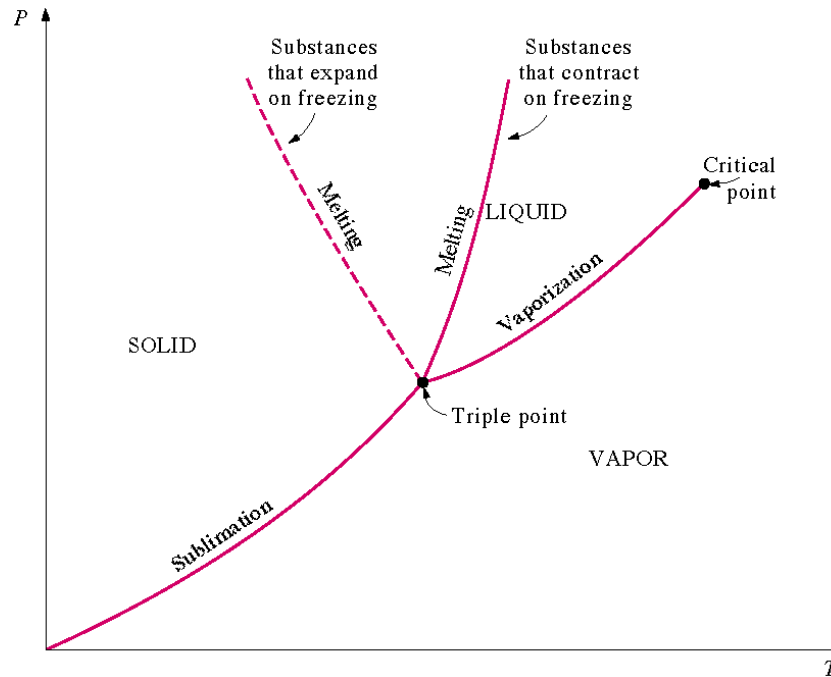


The triple point of water is 0.01°C , 0.6117 kPa (See Table 3-3).

The critical point of water is 373.95°C , 22.064 MPa (See Table A-1).

Plot the following processes on the P - T diagram for water (expands on freezing) and give examples of these processes from your personal experiences.

1. process a-b: liquid to vapor transition
2. process c-d: solid to liquid transition
3. process e-f: solid to vapor transition



Property Tables

In addition to the temperature, pressure, and volume data, Tables A-4 through A-8 contain the data for the specific internal energy u the specific enthalpy h and the specific entropy s . The enthalpy is a convenient grouping of the internal energy, pressure, and volume and is given by

$$H = U + PV$$

The enthalpy per unit mass is

$$h = u + Pv$$

We will find that the enthalpy h is quite useful in calculating the energy of mass streams flowing into and out of control volumes. The enthalpy is also useful in the energy balance during a constant pressure process for a substance contained in a closed piston-cylinder device. The enthalpy has units of energy per unit mass, kJ/kg. The entropy s is a property defined by the second law of thermodynamics and is related to the heat transfer to a system divided by the system temperature; thus, the entropy has units of energy divided by temperature. The concept of entropy is explained in Chapters 6 and 7.

Saturated Water Tables

Since temperature and pressure are dependent properties using the phase change, two tables are given for the saturation region. Table A-4 has temperature as the independent property; Table A-5 has pressure as the independent property. These two tables contain the same information and often only one table is given.

For the complete Table A-4, the last entry is the critical point at 373.95°C .

TABLE A-4

Saturated water-Temperature table

See next slide.

Temp., T °C	Sat. Press., P_{sat} kPa	Specific volume, m^3/kg		Internal energy, kJ/kg			Enthalpy, kJ/kg			Entropy, $kJ/kg\cdot K$		
		Sat. liquid, v_f	Sat. vapor, v_g	Sat. liquid, u_f	Evap., u_{fg}	Sat. vapor, u_g	Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g	Sat. liquid, s_f	Evap., s_{fg}	Sat. vapor, s_g
0.01	0.6117	0.001000	206.00	0.00	2374.9	2374.9	0.00	2500.9	2500.9	0.0000	9.1556	9.1556
5	0.8725	0.001000	147.03	21.02	2360.8	2381.8	21.02	2489.1	2510.1	0.0763	8.9487	9.0249
10	1.228	0.001000	106.32	42.02	2346.6	2388.7	42.02	2477.2	2519.2	0.1511	8.7488	8.8999
15	1.706	0.001001	77.885	62.98	2332.5	2395.5	62.98	2465.4	2528.3	0.2245	8.5559	8.7803
20	2.339	0.001002	57.762	83.91	2318.4	2402.3	83.91	2453.5	2537.4	0.2965	8.3696	8.6661
25	3.170	0.001003	43.340	104.83	2304.3	2409.1	104.83	2441.7	2546.5	0.3672	8.1895	8.5567
30	4.247	0.001004	32.879	125.73	2290.2	2415.9	125.74	2429.8	2555.6	0.4368	8.0152	8.4520
35	5.629	0.001006	25.205	146.63	2276.0	2422.7	146.64	2417.9	2564.6	0.5051	7.8466	8.3517
40	7.385	0.001008	19.515	167.53	2261.9	2429.4	167.53	2406.0	2573.5	0.5724	7.6832	8.2556
45	9.595	0.001010	15.251	188.43	2247.7	2436.1	188.44	2394.0	2582.4	0.6386	7.5247	8.1633
50	12.35	0.001012	12.026	209.33	2233.4	2442.7	209.34	2382.0	2591.3	0.7038	7.3710	8.0748
55	15.76	0.001015	9.5639	230.24	2219.1	2449.3	230.26	2369.8	2600.1	0.7680	7.2218	7.9898
60	19.95	0.001017	7.6670	251.16	2204.7	2455.9	251.18	2357.7	2608.8	0.8313	7.0769	7.9082
65	25.04	0.001020	6.1935	272.09	2190.3	2462.4	272.12	2345.4	2617.5	0.8937	6.9360	7.8296
70	31.20	0.001023	5.0396	293.04	2175.8	2468.9	293.07	2333.0	2626.1	0.9551	6.7989	7.7540
75	38.60	0.001026	4.1291	313.99	2161.3	2475.3	314.03	2320.6	2634.6	1.0158	6.6655	7.6812
80	47.42	0.001029	3.4053	334.97	2146.6	2481.6	335.02	2308.0	2643.0	1.0756	6.5355	7.6111
85	57.87	0.001032	2.8261	355.96	2131.9	2487.8	356.02	2295.3	2651.4	1.1346	6.4089	7.5435
90	70.18	0.001036	2.3593	376.97	2117.0	2494.0	377.04	2282.5	2659.6	1.1929	6.2853	7.4782
95	84.61	0.001040	1.9808	398.00	2102.0	2500.1	398.09	2269.6	2667.6	1.2504	6.1647	7.4151
100	101.42	0.001043	1.6720	419.06	2087.0	2506.0	419.17	2256.4	2675.6	1.3072	6.0470	7.3542
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360	18666	0.001895	0.006950	1726.16	625.7	2351.9	1761.53	720.1	2481.6	3.9165	1.1373	5.0537
365	19822	0.002015	0.006009	1777.22	526.4	2303.6	1817.16	605.5	2422.7	4.0004	0.9489	4.9493
370	21044	0.002217	0.004953	1844.53	385.6	2230.1	1891.19	443.1	2334.3	4.1119	0.6890	4.8009
373.95	22064	0.003106	0.003106	2015.8	0	2015.8	2084.3	0	2084.3	4.4070	0	4.4070

TABLE A-5

Saturated water-Pressure table

Press. P kPa	Sat. Temp., T_{sat} °C	Specific volume, m^3/kg			Internal energy, kJ/kg		Enthalpy, kJ/kg			Entropy, kJ/kg·K		
		Sat. liquid, v_f	Sat. vapor, v_g	Sat. liquid, u_f	Evap., u_{fg}	Sat. vapor, u_g	Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g	Sat. liquid, s_f	Evap., s_{fg}	Sat. vapor, s_g
0.6117	0.01	0.001000	206.00	0.00	2374.9	2374.9	0.00	2500.9	2500.9	0.0000	9.1556	9.1556
1.0	6.97	0.001000	129.19	29.30	2355.2	2384.5	29.30	2484.4	2513.7	0.1059	8.8690	8.9749
1.5	13.02	0.001001	87.964	54.69	2338.1	2392.8	54.69	2470.1	2524.7	0.1956	8.6314	8.8270
2.0	17.50	0.001001	66.990	73.43	2325.5	2398.9	73.43	2459.5	2532.9	0.2606	8.4621	8.7227
2.5	21.08	0.001002	54.242	88.42	2315.4	2403.8	88.42	2451.0	2539.4	0.3118	8.3302	8.6421
3.0	24.08	0.001003	45.654	100.98	2306.9	2407.9	100.98	2443.9	2544.8	0.3543	8.2222	8.5765
4.0	28.96	0.001004	34.791	121.39	2293.1	2414.5	121.39	2432.3	2553.7	0.4224	8.0510	8.4734
5.0	32.87	0.001005	28.185	137.75	2282.1	2419.8	137.75	2423.0	2560.7	0.4762	7.9176	8.3938
7.5	40.29	0.001008	19.233	168.74	2261.1	2429.8	168.75	2405.3	2574.0	0.5763	7.6738	8.2501
10	45.81	0.001010	14.670	191.79	2245.4	2437.2	191.81	2392.1	2583.9	0.6492	7.4996	8.1488
15	53.97	0.001014	10.020	225.93	2222.1	2448.0	225.94	2372.3	2598.3	0.7549	7.2522	8.0071
20	60.06	0.001017	7.6481	251.40	2204.6	2456.0	251.42	2357.5	2608.9	0.8320	7.0752	7.9073
25	64.96	0.001020	6.2034	271.93	2190.4	2462.4	271.96	2345.5	2617.5	0.8932	6.9370	7.8302
30	69.09	0.001022	5.2287	289.24	2178.5	2467.7	289.27	2335.3	2624.6	0.9441	6.8234	7.7675
40	75.86	0.001026	3.9933	317.58	2158.8	2476.3	317.62	2318.4	2636.1	1.0261	6.6430	7.6691
50	81.32	0.001030	3.2403	340.49	2142.7	2483.2	340.54	2304.7	2645.2	1.0912	6.5019	7.5931
75	91.76	0.001037	2.2172	384.36	2111.8	2496.1	384.44	2278.0	2662.4	1.2132	6.2426	7.4558
100	99.61	0.001043	1.6941	417.40	2088.2	2505.6	417.51	2257.5	2675.0	1.3028	6.0562	7.3589
125	105.97	0.001048	1.3750	444.23	2068.8	2513.0	444.36	2240.6	2684.9	1.3741	5.9100	7.2841
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20,000	365.75	0.002038	0.005862	1785.84	509.0	2294.8	1826.59	585.5	2412.1	4.0146	0.9164	4.9310
21,000	369.83	0.002207	0.004994	1841.62	391.9	2233.5	1887.97	450.4	2338.4	4.1071	0.7005	4.8076
22,000	373.71	0.002703	0.003644	1951.65	140.8	2092.4	2011.12	161.5	2172.6	4.2942	0.2496	4.5419
22,064	373.95	0.003106	0.003106	2015.8	0	2015.8	2084.3	0	2084.3	4.4070	0	4.4070

For the complete Table A-5, the last entry is the critical point at 22.064 MPa.

Saturation pressure is the pressure at which the liquid and vapor phases are in equilibrium at a given temperature.

Saturation temperature is the temperature at which the liquid and vapor phases are in equilibrium at a given pressure.

In Figure 3-11, states 2, 3, and 4 are saturation states.

The subscript **fg** used in Tables A-4 and A-5 refers to the difference between the saturated vapor value and the saturated liquid value region. That is,

$$u_{fg} = u_g - u_f$$

$$h_{fg} = h_g - h_f$$

$$s_{fg} = s_g - s_f$$

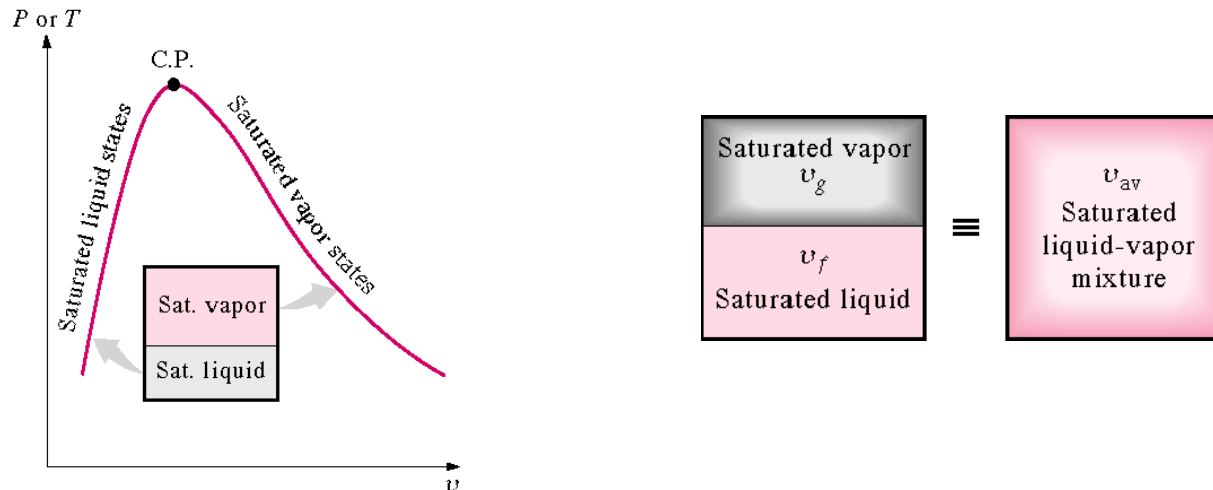
The quantity h_{fg} is called the enthalpy of vaporization (or latent heat of vaporization). It represents the amount of energy needed to vaporize a unit of mass of saturated liquid at a given temperature or pressure. It decreases as the temperature or pressure increases, and becomes zero at the critical point.

Quality and Saturated Liquid-Vapor Mixture

Now, let's review the constant pressure heat addition process for water shown in Figure 3-11. Since state 3 is a mixture of saturated liquid and saturated vapor, how do we locate it on the T-v diagram? To establish the location of state 3 a new parameter called the quality x is defined as

$$x = \frac{\text{mass}_{\text{saturated vapor}}}{\text{mass}_{\text{total}}} = \frac{m_g}{m_f + m_g}$$

The quality is zero for the saturated liquid and one for the saturated vapor ($0 \leq x \leq 1$). The average specific volume at any state 3 is given in terms of the quality as follows. Consider a mixture of saturated liquid and saturated vapor. The liquid has a mass m_f and occupies a volume V_f . The vapor has a mass m_g and occupies a volume V_g .



We note

$$V = V_f + V_g$$

$$m = m_f + m_g$$

$$V = mv, \quad V_f = m_f v_f, \quad V_g = m_g v_g$$

$$mv = m_f v_f + m_g v_g$$

$$v = \frac{m_f v_f}{m} + \frac{m_g v_g}{m}$$

Recall the definition of quality x

$$x = \frac{m_g}{m} = \frac{m_g}{m_f + m_g}$$

Then

$$\frac{m_f}{m} = \frac{m - m_g}{m} = 1 - x$$

Note, quantity $1 - x$ is often given the name moisture. The specific volume of the saturated mixture becomes

$$v = (1 - x)v_f + xv_g$$

The form that we use most often is

$$v = v_f + x(v_g - v_f)$$

It is noted that the value of any extensive property per unit mass in the saturation region is calculated from an equation having a form similar to that of the above equation. Let Y be any extensive property and let y be the corresponding intensive property, Y/m , then

$$\begin{aligned} y &= \frac{Y}{m} = y_f + x(y_g - y_f) \\ &= y_f + x y_{fg} \end{aligned}$$

$$\text{where } y_{fg} = y_g - y_f$$

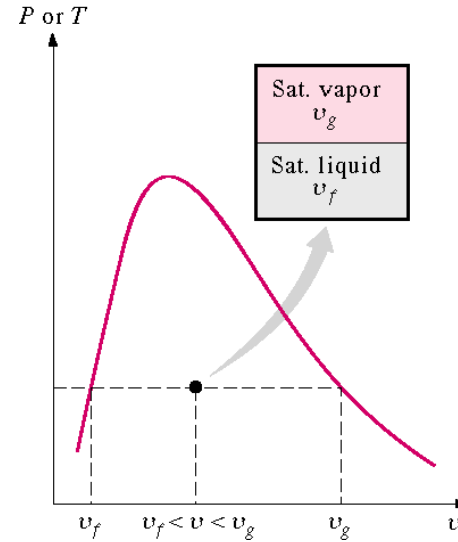
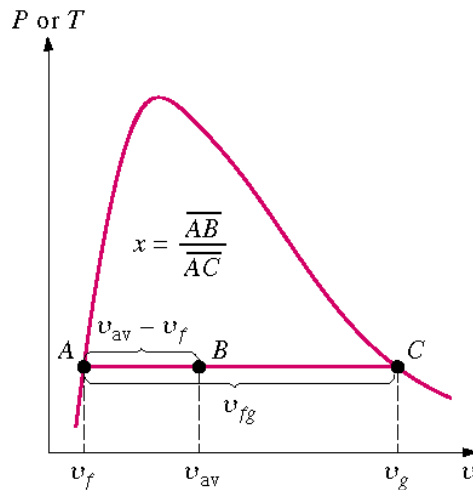
The term y_{fg} is the difference between the saturated vapor and the saturated liquid values of the property y ; y may be replaced by any of the variables v , u , h , or s .

We often use the above equation to determine the quality x of a saturated liquid-vapor state.

The following application is called the Lever Rule:

$$x = \frac{y - y_f}{y_{fg}}$$

The Lever Rule is illustrated in the following figures.



Superheated Water Table

A substance is said to be **superheated** if the given temperature is greater than the saturation temperature for the given pressure.

State 5 in Figure 3-11 is a superheated state.

In the superheated water Table A-6, T and P are the independent properties. The value of temperature to the right of the pressure is the saturation temperature for the pressure.

The first entry in the table is the saturated vapor state at the pressure.

TABLE A-6 Superheated water

T °C	v m ³ /kg	u kJ/kg	h kJ/kg	s kJ/kg·K
$P = 0.01 \text{ MPa (45.81}^\circ\text{C)}$				
Sat.	14.670	2437.2	2583.9	8.1488
50	14.867	2443.3	2592.0	8.1741
100	17.196	2515.5	2687.5	8.4489
150	19.513	2587.9	2783.0	8.6893
200	21.826	2661.4	2879.6	8.9049
250	24.136	2736.1	2977.5	9.1015
300	26.446	2812.3	3076.7	9.2827
400	31.063	2969.3	3280.0	9.6094
500	35.680	3132.9	3489.7	9.8998
600	40.296	3303.3	3706.3	10.1631
700	44.911	3480.8	3929.9	10.4056
800	49.527	3665.4	4160.6	10.6312
900	54.143	3856.9	4398.3	10.8429
1000	58.758	4055.3	4642.8	11.0429
1100	63.373	4260.0	4893.8	11.2326
1200	67.989	4470.9	5150.8	11.4132
1300	72.604	4687.4	5413.4	11.5857

T °C	v m ³ /kg	u kJ/kg	h kJ/kg	s kJ/kg·K
$P = 0.05 \text{ MPa (81.32}^\circ\text{C)}$				
Sat.	3.2403	2483.2	2645.2	7.5931
100	3.4187	2511.5	2682.4	7.6953
150	3.8897	2585.7	2780.2	7.9413
200	4.3562	2660.0	2877.8	8.1592
250	4.8206	2735.1	2976.2	8.3568
300	5.2841	2811.6	3075.8	8.5387
400	6.2094	2968.9	3279.3	8.8659
500	7.1338	3132.6	3489.3	9.1566
600	8.0577	3303.1	3706.0	9.4201
700	8.9813	3480.6	3929.7	9.6626
800	9.9047	3665.2	4160.4	9.8883
900	10.828	3856.8	4398.2	10.1000
1000	11.751	4055.2	4642.7	10.3000
1100	12.675	4259.9	4893.7	10.4897
1200	13.598	4470.8	5150.7	10.6704
1300	14.521	4687.3	5413.3	10.8429

Compressed Liquid Water Table

A substance is said to be a **compressed liquid** when the pressure is greater than the saturation pressure for the temperature.

It is now noted that state 1 in Figure 3-11 is called a compressed liquid state because the saturation pressure for the temperature T_1 is less than P_1 .

Data for water compressed liquid states are found in the compressed liquid tables, Table A-7. Table A-7 is arranged like Table A-6, except the saturation states are the saturated liquid states. Note that the data in Table A-7 begins at 5 MPa or 50 times atmospheric pressure.

TABLE A-7 Compressed liquid water

T °C	v m ³ /kg	u kJ/kg	h kJ/kg	s kJ/kg·K
$P = 5 \text{ MPa (263.94}^\circ\text{C)}$				
Sat.	0.0012862	1148.1	1154.5	2.9207
0	0.0009977	0.04	5.03	0.0001
20	0.0009996	83.61	88.61	0.2954
40	0.0010057	166.92	171.95	0.5705
60	0.0010149	250.29	255.36	0.8287
80	0.0010267	333.82	338.96	1.0723
100	0.0010410	417.65	422.85	1.3034
120	0.0010576	501.91	507.19	1.5236
140	0.0010769	586.80	592.18	1.7344
160	0.0010988	672.55	678.04	1.9374
180	0.0011240	759.47	765.09	2.1338
200	0.0011531	847.92	853.68	2.3251
220	0.0011868	938.39	944.32	2.5127
240	0.0012268	1031.6	1037.7	2.6983
260	0.0012755	1128.5	1134.9	2.8841

T °C	v m ³ /kg	u kJ/kg	h kJ/kg	s kJ/kg·K
$P = 10 \text{ MPa (311.00}^\circ\text{C)}$				
Sat.	0.0014522	1393.3	1407.9	3.3603
0	0.0009952	0.12	10.07	0.0003
20	0.0009973	83.31	93.28	0.2943
40	0.0010035	166.33	176.37	0.5685
60	0.0010127	249.43	259.55	0.8260
80	0.0010244	332.69	342.94	1.0691
100	0.0010385	416.23	426.62	1.2996
120	0.0010549	500.18	510.73	1.5191
140	0.0010738	584.72	595.45	1.7293
160	0.0010954	670.06	681.01	1.9316
180	0.0011200	756.48	767.68	2.1271
200	0.0011482	844.32	855.80	2.3174
220	0.0011809	934.01	945.82	2.5037
240	0.0012192	1026.2	1038.3	2.6876
260	0.0012653	1121.6	1134.3	2.8710
280	0.0013226	1221.8	1235.0	3.0565
300	0.0013980	1329.4	1343.3	3.2488

At pressures below 5 MPa for water, the data are approximately equal to the saturated liquid data at the given **temperature**. We approximate intensive parameter y , that is v , u , h , and s data as

$$y \cong y_f @ T$$

The enthalpy is more sensitive to variations in pressure; therefore, at high pressures the enthalpy can be approximated by

$$h \cong h_f @ T + v_f (P - P_{sat})$$

For our work, the compressed liquid enthalpy may be approximated by

$$h \cong h_f @ T$$

Saturated Ice-Water Vapor Table

When the temperature of a substance is below the triple point temperature, the saturated solid and liquid phases exist in equilibrium. Here we define the quality as the ratio of the mass that is vapor to the total mass of solid and vapor in the saturated solid-vapor mixture. The process of changing directly from the solid phase to the vapor phase is called sublimation. Data for saturated ice and water vapor are given in Table A-8. In Table A-8, the term **Subl.** refers to the difference between the saturated vapor value and the saturated solid value.

TABLE A-8
Saturated ice-water vapor

Temp., T °C	Sat. Press., P_{sat} kPa	Specific volume, m^3/kg		Internal energy, kJ/kg			Enthalpy, kJ/kg			Entropy, kJ/kg·K		
		Sat. ice, v_i	Sat. vapor, v_g	Sat. ice, u_i	Subl., u_{ig}	Sat. vapor, u_g	Sat. ice, h_i	Subl., h_{ig}	Sat. vapor, h_g	Sat. ice, s_i	Subl., s_{ig}	Sat. vapor, s_g
0.01	0.6117	0.0010909	206.0	-333.40		2374.5	-333.40		2500.5	-1.220		9.154
0	0.6112	0.0010909	206.2	-333.43	2707.9	2374.5	-333.43	2833.9	2500.5	-1.220	10.375	9.154
-2	0.5177	0.0010905	241.6	-337.63	2709.4	2371.8	-337.63	2834.5	2496.8	-1.236	10.453	9.218
-4	0.4375	0.0010902	283.8	-341.80	2710.8	2369.0	-341.80	2835.0	2493.2	-1.251	10.533	9.282
-6	0.3687	0.0010898	334.3	-345.94	2712.2	2366.2	-345.93	2835.4	2489.5	-1.267	10.613	9.347
-8	0.3100	0.0010895	394.7	-350.04	2713.5	2363.5	-350.04	2835.8	2485.8	-1.282	10.695	9.413
-10	0.2599	0.0010892	467.2	-354.12	2714.8	2360.7	-354.12	2836.2	2482.1	-1.298	10.778	9.480
.
.
-36	0.0200	0.0010850	5460.1	-404.40	2729.0	2324.6	-404.40	2838.4	2434.0	-1.499	11.969	10.470
-38	0.0161	0.0010847	6750.5	-408.07	2729.9	2321.8	-408.07	2838.4	2430.3	-1.514	12.071	10.557
-40	0.0128	0.0010844	8376.7	-411.70	2730.7	2319.0	-411.70	2838.3	2426.6	-1.530	12.174	10.644

The specific volume, internal energy, enthalpy, and entropy for a mixture of saturated ice and saturated vapor are calculated similarly to that of saturated liquid-vapor mixtures.

$$y_{ig} = y_g - y_i$$

$$y = y_i + x y_{ig}$$

where the quality x of a saturated ice-vapor state is

$$x = \frac{m_g}{m_i + m_g}$$

How to Choose the Right Table

The correct table to use to find the thermodynamic properties of a real substance can always be determined by comparing the known state properties to the properties in the saturation region. Given the temperature or pressure and one other property from the group v , u , h , and s , the following procedure is used. For example if the pressure and specific volume are specified, three questions are asked. For the given pressure,

For the given pressure

Is $v < v_f$?

Is $v_f < v < v_g$?

Is $v_g < v$?

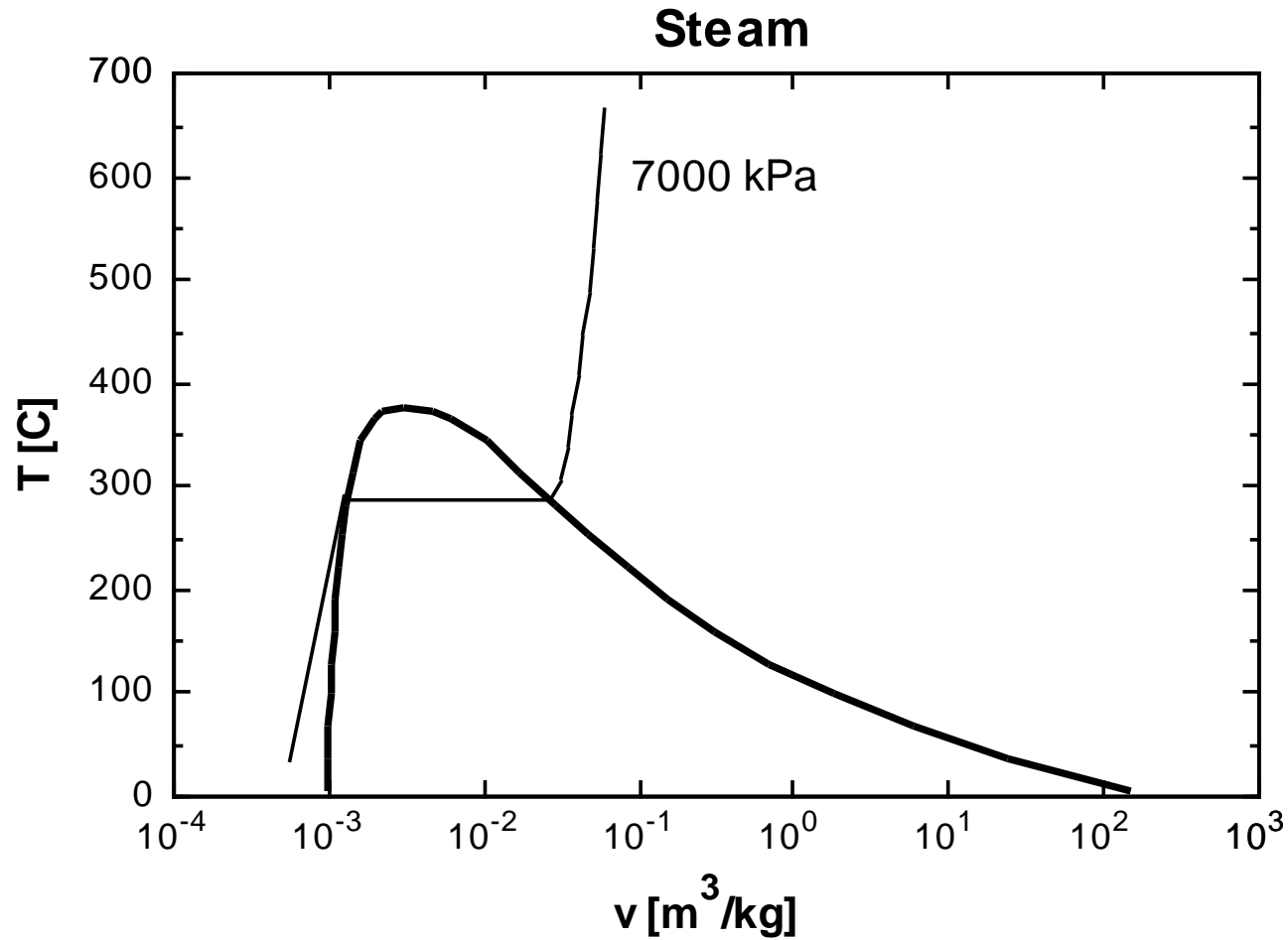
The answer to one of these questions must be yes. If the answer to the first question is yes, the state is in the compressed liquid region, and the compressed liquid tables are used to find the properties of the state. If the answer to the second question is yes, the state is in the saturation region, and either the saturation temperature table or the saturation pressure table is used to find the properties. Then the quality is calculated and is used to calculate the other properties, u , h , and s . If the answer to the third question is yes, the state is in the superheated region and the superheated tables are used to find the other properties.

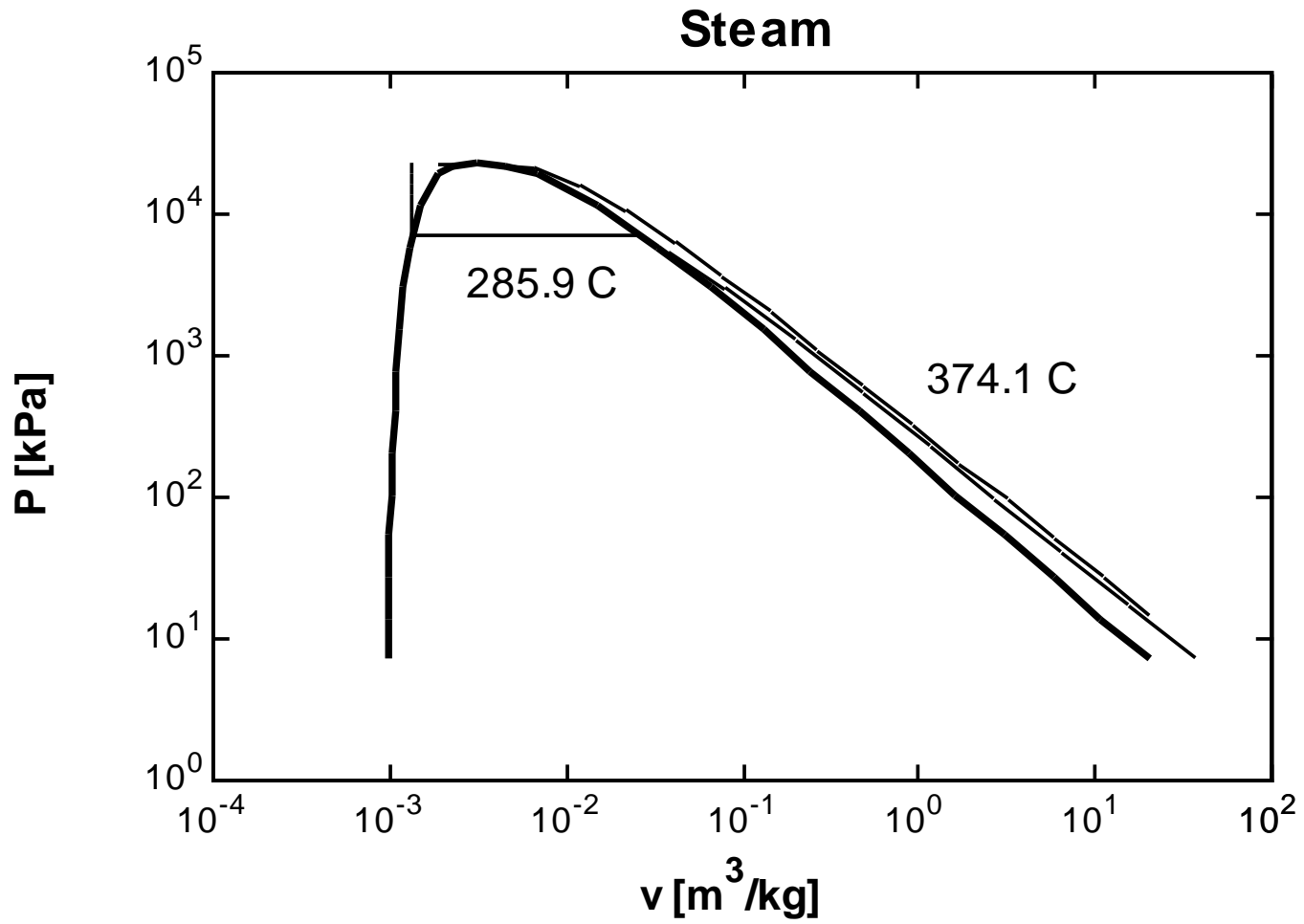
Some tables may not always give the internal energy. When it is not listed, the internal energy is calculated from the definition of the enthalpy as

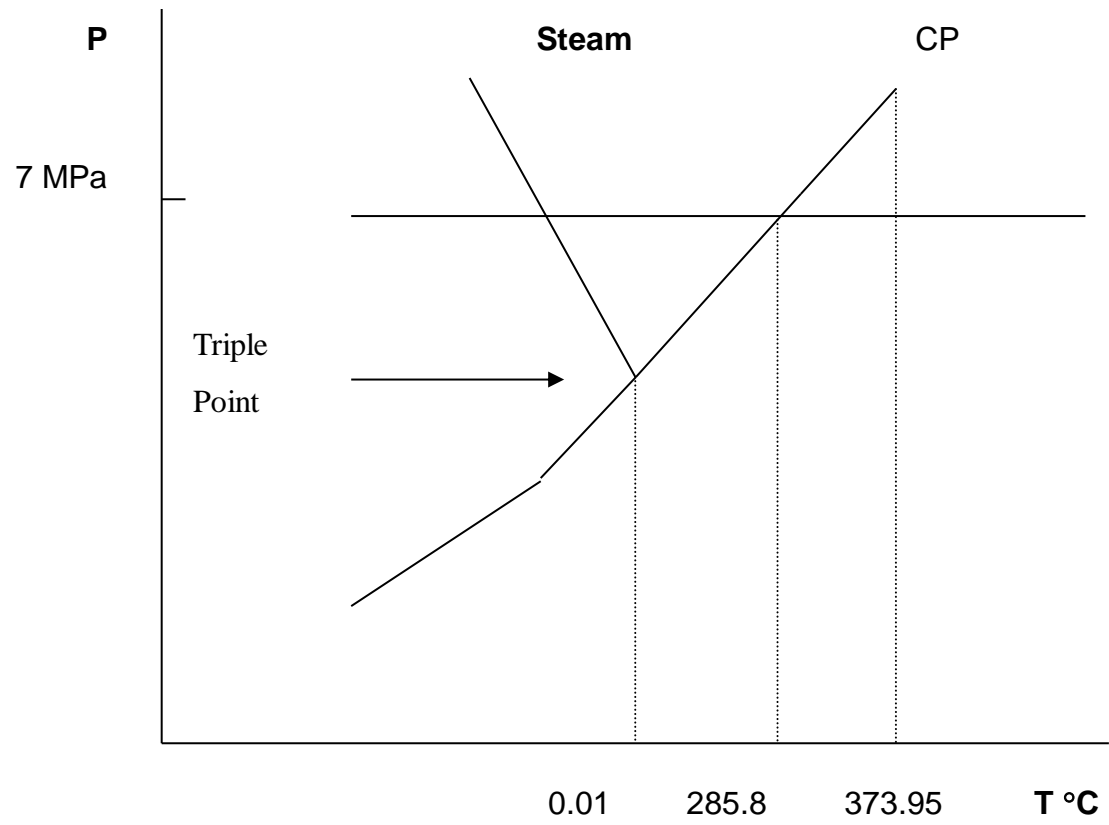
$$u = h - Pv$$

Example 3-1

Find the internal energy of water at the given states for 7 MPa and plot the states on T - v , P - v , and P - T diagrams.







1. $P = 7$ MPa, dry saturated or saturated vapor

Using Table A-5,

$$u = u_g = 2581.0 \frac{\text{kJ}}{\text{kg}}$$

Locate state 1 on the T - v , P - v , and P - T diagrams.

2. $P = 7$ MPa, wet saturated or saturated liquid

Using Table A-5,

$$u = u_f = 1258.0 \frac{\text{kJ}}{\text{kg}}$$

Locate state 2 on the T - v , P - v , and P - T diagrams.

3. Moisture = 5%, $P = 7$ MPa

let moisture be y , defined as

$$y = \frac{m_f}{m} = 0.05$$

then, the quality is

$$x = 1 - y = 1 - 0.05 = 0.95$$

and using Table A-5,

$$\begin{aligned}u &= u_f + x(u_g - u_f) \\ &= 1258.0 + 0.95(2581.0 - 1257.6) \\ &= 2514.4 \frac{\text{kJ}}{\text{kg}}\end{aligned}$$

Notice that we could have used

$$u = u_f + x u_{fg}$$

Locate state 3 on the T - v , P - v , and P - T diagrams.

4. $P = 7 \text{ MPa}$, $T = 600^\circ\text{C}$

For $P = 7 \text{ MPa}$, Table A-5 gives $T_{\text{sat}} = 285.83^\circ\text{C}$. Since $600^\circ\text{C} > T_{\text{sat}}$ for this pressure, the state is superheated. Use Table A-6.

$$u = 3261.0 \frac{\text{kJ}}{\text{kg}}$$

Locate state 4 on the T - v , P - v , and P - T diagrams.

5. $P = 7 \text{ MPa}$, $T = 100^\circ\text{C}$

Using Table A-4, At $T = 100^\circ\text{C}$, $P_{\text{sat}} = 0.10142 \text{ MPa}$. Since $P > P_{\text{sat}}$, the state is compressed liquid.

Approximate solution:

$$u \cong u_{f@T=100\text{C}} = 419.06 \frac{\text{kJ}}{\text{kg}}$$

Solution using Table A-7:

We do linear interpolation to get the value at 100°C . (We will demonstrate how to do linear interpolation with this problem even though one could accurately estimate the answer.)

$P \text{ MPa}$	$u \text{ kJ/kg}$
5	417.65
7	$u = ?$
10	416.23

The interpolation scheme is called “the ratio of corresponding differences.”

Using the above table, form the following ratios.

$$\frac{5-7}{5-10} = \frac{417.65-u}{417.65-416.23}$$

$$u = 417.08 \frac{\text{kJ}}{\text{kg}}$$

Locate state 5 on the T - v , P - v , and P - T diagrams.

6. $P = 7 \text{ MPa}$, $T = 460^\circ\text{C}$

Since $460^\circ\text{C} > T_{\text{sat}} = 385.83^\circ\text{C}$ at $P = 7 \text{ MPa}$, the state is superheated. Using Table A-6, we do a linear interpolation to calculate u .

T °C	u kJ/kg
450	2979.0
460	$u = ?$
500	3074.3

Using the above table, form the following ratios.

$$\frac{460 - 450}{500 - 450} = \frac{u - 2979.0}{3074.3 - 2979.0}$$

$$u = 2998.1 \frac{\text{kJ}}{\text{kg}}$$

Locate state 6 on the T - v , P - v , and P - T diagrams.

Example 3-2

Determine the enthalpy of 1.5 kg of water contained in a volume of 1.2 m³ at 200 kPa.

Recall we need two independent, intensive properties to specify the state of a simple substance. Pressure P is one intensive property and specific volume is another. Therefore, we calculate the specific volume.

$$v = \frac{\text{Volume}}{\text{mass}} = \frac{1.2 \text{ m}^3}{1.5 \text{ kg}} = 0.8 \frac{\text{m}^3}{\text{kg}}$$

Using Table A-5 at $P = 200$ kPa,

$$v_f = 0.001061 \text{ m}^3/\text{kg}, \quad v_g = 0.8858 \text{ m}^3/\text{kg}$$

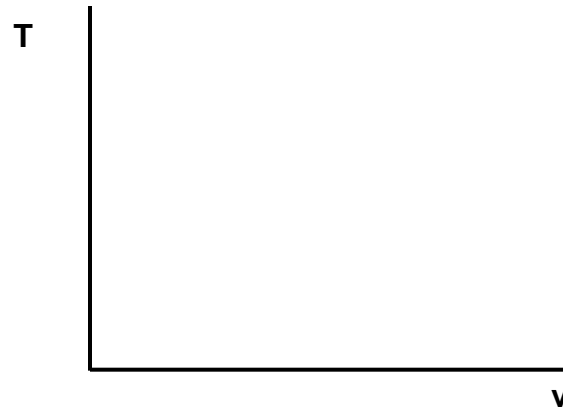
Now,

Is $v < v_f$? No

Is $v_f < v < v_g$? Yes

Is $v_g < v$? No

Locate this state on a T - v diagram.



We see that the state is in the two-phase or saturation region. So we must find the quality x first.

$$v = v_f + x(v_g - v_f)$$

$$\begin{aligned}
 x &= \frac{v - v_f}{v_g - v_f} \\
 &= \frac{0.8 - 0.001061}{0.8858 - 0.001061} \\
 &= 0.903 \quad (\text{What does this mean?})
 \end{aligned}$$

Then,

$$\begin{aligned}
 h &= h_f + x h_{fg} \\
 &= 504.7 + (0.903)(2201.6) \\
 &= 2492.7 \frac{\text{kJ}}{\text{kg}}
 \end{aligned}$$

Example 2-3

Determine the internal energy of refrigerant-134a at a temperature of 0°C and a quality of 60%.

Using Table A-11, for $T = 0^\circ\text{C}$,

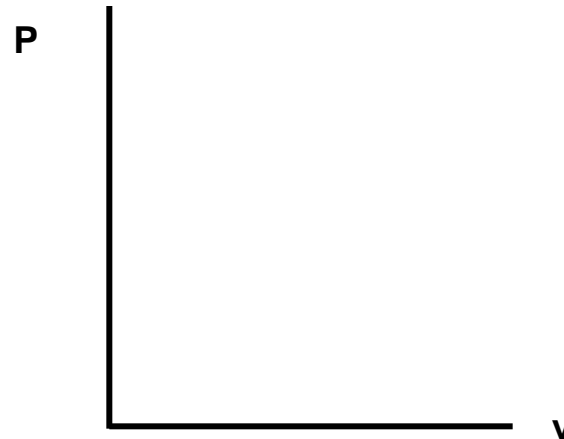
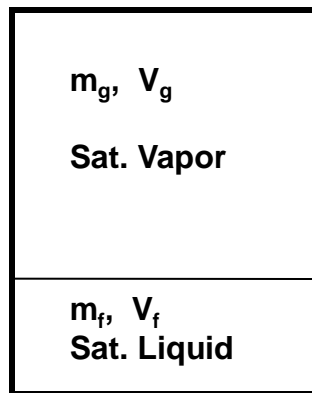
$$u_f = 51.63 \text{ kJ/kg} \quad u_g = 230.16 \text{ kJ/kg}$$

then,

$$\begin{aligned}u &= u_f + x(u_g - u_f) \\ &= 51.63 + (0.6)(230.16 - 51.63) \\ &= 158.75 \frac{\text{kJ}}{\text{kg}}\end{aligned}$$

Example 2-4

Consider the closed, rigid container of water shown below. The pressure is 700 kPa, the mass of the saturated liquid is 1.78 kg, and the mass of the saturated vapor is 0.22 kg. Heat is added to the water until the pressure increases to 8 MPa. Find the final temperature, enthalpy, and internal energy of the water. Does the liquid level rise or fall? Plot this process on a P - v diagram with respect to the saturation lines and the critical point.



Let's introduce a solution procedure that we will follow throughout the course. A similar solution technique is discussed in detail in Chapter 1.

System: A closed system composed of the water enclosed in the tank

Property Relation: Steam Tables

Process: Volume is constant (rigid container)

For the closed system the total mass is constant and since the process is one in which the volume is constant, the average specific volume of the saturated mixture during the process is given by

$$v = \frac{V}{m} = \text{constant}$$

or

$$v_2 = v_1$$

Now to find v_1 recall that in the two-phase region at state 1

$$x_1 = \frac{m_{g1}}{m_{f1} + m_{g1}} = \frac{0.22 \text{ kg}}{(1.78 + 0.22) \text{ kg}} = 0.11$$

Then, at $P = 700 \text{ kPa}$

$$\begin{aligned} v_1 &= v_{f1} + x_1 (v_{g1} - v_{f1}) \\ &= 0.001108 + (0.11)(0.2728 - 0.001108) \\ &= 0.031 \frac{\text{m}^3}{\text{kg}} \end{aligned}$$

State 2 is specified by:

$$P_2 = 8 \text{ MPa}, \quad v_2 = 0.031 \text{ m}^3/\text{kg}$$

At $8 \text{ MPa} = 8000 \text{ kPa}$,

$$v_f = 0.001384 \text{ m}^3/\text{kg} \quad v_g = 0.02352 \text{ m}^3/\text{kg}$$

at 8 MPa , $v_2 = 0.031 \text{ m}^3/\text{kg}$.

Is $v_2 < v_f$? No

Is $v_f < v_2 < v_g$? No

Is $v_g < v_2$? Yes

Therefore, State 2 is superheated.

Interpolating in the superheated tables at 8 MPa, $v = 0.031 \text{ m}^3/\text{kg}$ gives,

$$T_2 = 361 \text{ }^\circ\text{C}$$

$$h_2 = 3024 \text{ kJ/kg}$$

$$u_2 = 2776 \text{ kJ/kg}$$

Since state 2 is superheated, the liquid level falls.

Extra Problem

What would happen to the liquid level in the last example if the specific volume had been $0.001 \text{ m}^3/\text{kg}$ and the pressure was 8 MPa?

For more information on how steam is used in industry, check out this site:

www.eere.energy.gov/industry/bestpractices/steam.html

Equations of State

The relationship among the state variables, temperature, pressure, and specific volume is called the equation of state. We now consider the equation of state for the vapor or gaseous phase of simple compressible substances.

$$F(P, T, \nu) \equiv 0$$

Ideal Gas

Based on our experience in chemistry and physics we recall that the combination of Boyle's and Charles' laws for gases at low pressure result in the equation of state for the ideal gas as

$$P = R \left(\frac{T}{\nu} \right)$$

where R is the constant of proportionality and is called the **gas constant** and takes on a different value for each gas. If a gas obeys this relation, it is called an ideal gas. We often write this equation as

$$P\nu = RT$$

The gas constant for ideal gases is related to the universal gas constant valid for all substances through the molar mass (or molecular weight). Let R_u be the universal gas constant. Then,

$$R = \frac{R_u}{M}$$

The mass, m , is related to the moles, N , of substance through the molecular weight or molar mass, M , see Table A-1. The molar mass is the ratio of mass to moles and has the same value regardless of the system of units.

$$M_{air} = 28.97 \frac{g}{gmol} = 28.97 \frac{kg}{kmol} = 28.97 \frac{lbm}{lbmol}$$

Since 1 kmol = 1000 gmol or 1000 gram-mole and 1 kg = 1000 g, 1 kmol of air has a mass of 28.97 kg or 28,970 grams.

$$m = N M$$

The ideal gas equation of state may be written several ways.

$$Pv = RT$$

$$P \frac{V}{m} = RT$$

$$PV = mRT$$

$$PV = \frac{m}{M} (MR)T$$

$$PV = NR_u T$$

$$P \frac{V}{N} = R_u T$$

$$P\bar{v} = R_u T$$

Here

P = **absolute pressure** in MPa, or kPa

\bar{v} = molar specific volume in m³/kmol

T = **absolute temperature** in K

R_u = 8.314 kJ/(kmol·K)

Some values of the universal gas constant are

Universal Gas Constant, R_u

8.314 kJ/(kmol·K)

8.314 kPa·m³/(kmol·K)

1.986 Btu/(lbmol·R)

1545 ft·lbf/(lbmol·R)

10.73 psia·ft³/(lbmol·R)

The ideal gas equation of state can be derived from basic principles if one assumes

1. Intermolecular forces are small.
2. Volume occupied by the particles is small.

Example 2-5

Determine the particular gas constant for air and hydrogen.

$$R = \frac{R_u}{M}$$

$$R_{air} = \frac{8.314 \frac{kJ}{kmol - K}}{28.97 \frac{kg}{kmol}} = 0.287 \frac{kJ}{kg - K}$$

$$R_{hydrogen} = \frac{8.314 \frac{kJ}{kmol - K}}{2.016 \frac{kg}{kmol}} = 4.124 \frac{kJ}{kg - K}$$

The ideal gas equation of state is used when (1) the pressure is small compared to the critical pressure or (2) when the temperature is twice the critical temperature and the pressure is less than 10 times the critical pressure. The critical point is that state where there is an instantaneous change from the liquid phase to the vapor phase for a substance. Critical point data are given in Table A-1.

Compressibility Factor

To understand the above criteria and to determine how much the ideal gas equation of state deviates from the actual gas behavior, we introduce the compressibility factor Z as follows.

or

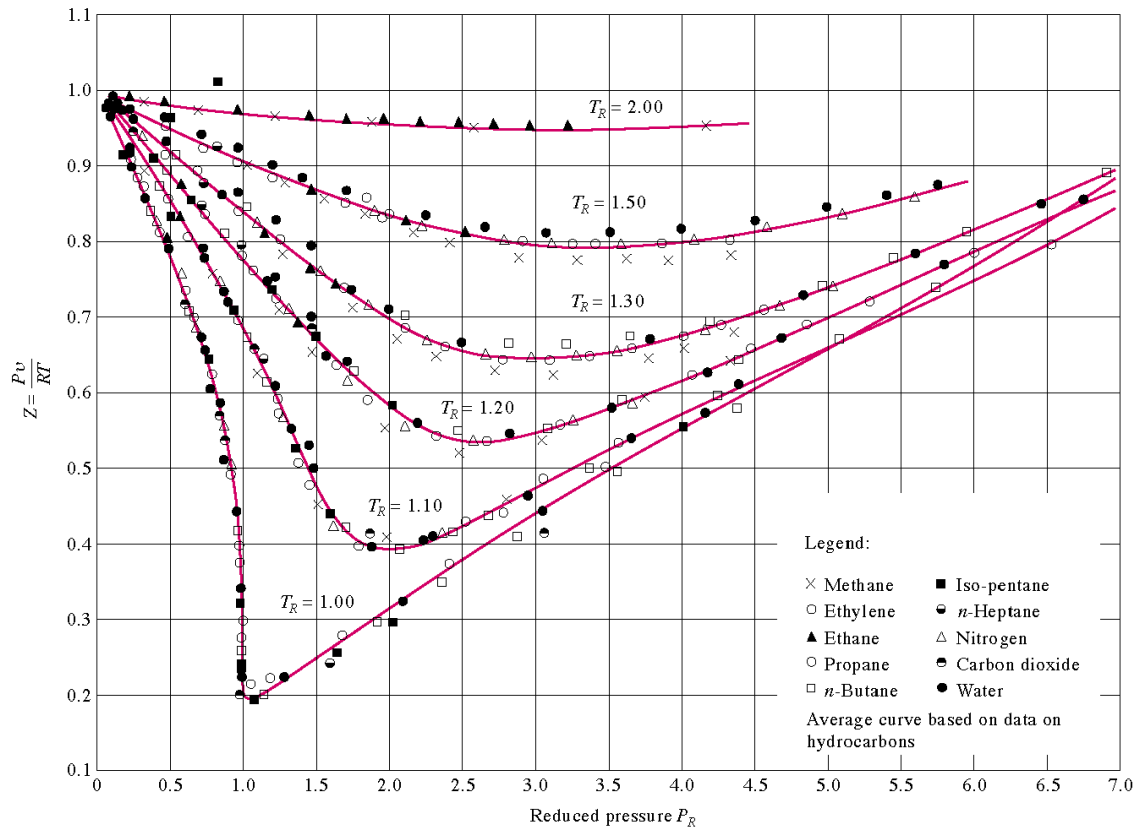
$$P\bar{v} = Z R_u T$$

$$Z = \frac{P\bar{v}}{R_u T}$$

For an ideal gas $Z = 1$, and the deviation of Z from unity measures the deviation of the actual P - V - T relation from the ideal gas equation of state. The compressibility factor is expressed as a function of the reduced pressure and the reduced temperature. The Z factor is approximately the same for all gases at the same *reduced temperature* and *reduced pressure*, which are defined as

$$T_R = \frac{T}{T_{cr}} \quad \text{and} \quad P_R = \frac{P}{P_{cr}}$$

where P_{cr} and T_{cr} are the critical pressure and temperature, respectively. The critical constant data for various substances are given in Table A-1. This is known as the *principle of corresponding states*. Figure 3-51 gives a comparison of Z factors for various gases and supports the principle of corresponding states.



When either P or T is unknown, Z can be determined from the compressibility chart with the help of the *pseudo-reduced specific volume*, defined as

$$v_R = \frac{v_{actual}}{RT_{cr}} \frac{P_{cr}}{P}$$

Figure A-15 presents the generalized compressibility chart based on data for a large number of gases.

These charts show the conditions for which $Z = 1$ and the gas behaves as an ideal gas:

1. $P_R < 10$ and $T_R > 2$ or $P < 10P_{cr}$ and $T > 2T_{cr}$
2. $P_R \ll 1$ or $P \ll P_{cr}$

Note: When P_R is small, we must make sure that the state is not in the compressed liquid region for the given temperature. A compressed liquid state is certainly not an ideal gas state.

For instance the critical pressure and temperature for oxygen are 5.08 MPa and 154.8 K, respectively. For temperatures greater than 300 K and pressures less than 50 MPa (1 atmosphere pressure is 0.10135 MPa) oxygen is considered to be an ideal gas.

Example 2-6

Calculate the specific volume of nitrogen at 300 K and 8.0 MPa and compare the result with the value given in a nitrogen table as $v = 0.011133 \text{ m}^3/\text{kg}$.

From Table A.1 for nitrogen

$$T_{cr} = 126.2 \text{ K}, P_{cr} = 3.39 \text{ MPa} \quad R = 0.2968 \text{ kJ/kg-K}$$

$$T_R = \frac{T}{T_{cr}} = \frac{300\text{K}}{126.2\text{K}} = 2.38$$

$$P_R = \frac{P}{P_{cr}} = \frac{8.0\text{MPa}}{3.39\text{MPa}} = 2.36$$

Since $T > 2T_{cr}$ and $P < 10P_{cr}$, we use the ideal gas equation of state

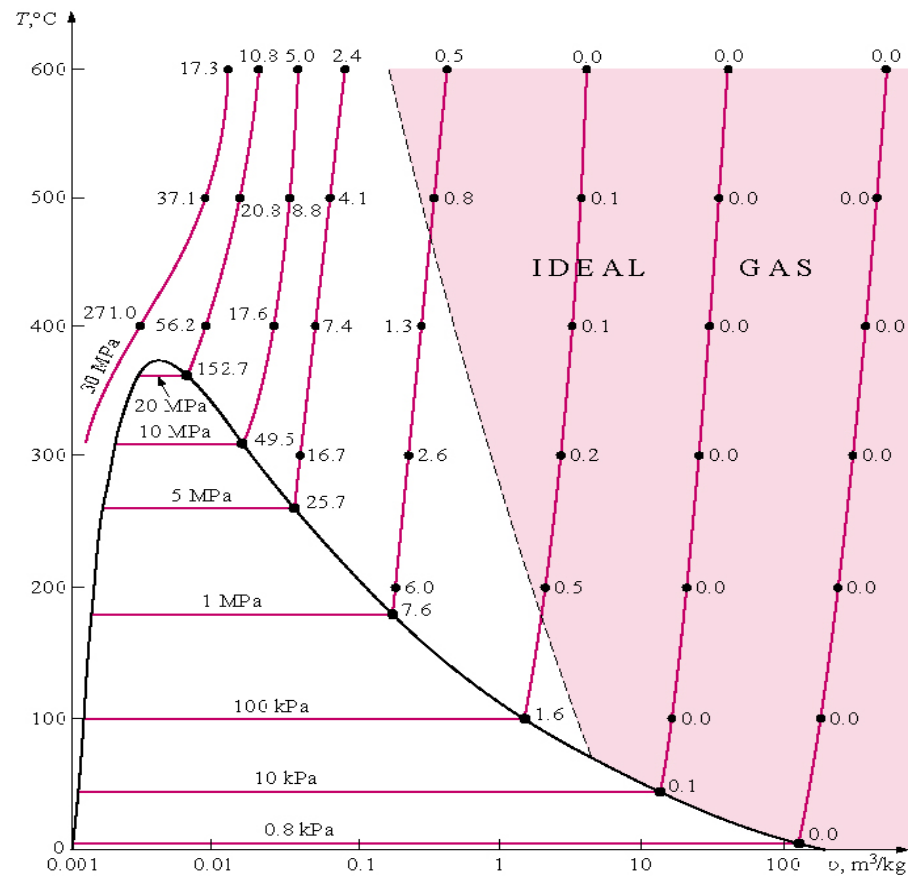
$$Pv = RT$$

$$v = \frac{RT}{P} = \frac{0.2968 \frac{\text{kJ}}{\text{kg-K}} (300\text{K})}{8.0\text{MPa}} \frac{\text{m}^3 \text{MPa}}{10^3 \text{kJ}}$$

$$= 0.01113 \frac{\text{m}^3}{\text{kg}}$$

Nitrogen is clearly an ideal gas at this state.

If the system pressure is low enough and the temperature high enough (P and T are compared to the critical values), gases will behave as ideal gases. Consider the T - v diagram for water. The figure below shows the percentage of error for the volume ($[\frac{v_{table} - v_{ideal}}{v_{table}}] \times 100$) for assuming water (superheated steam) to be an ideal gas.



We see that the region for which water behaves as an ideal gas is in the superheated region and depends on both T and P . We must be cautioned that in this course, when water is the working fluid, the ideal gas assumption may not be used to solve problems. We must use the real gas relations, i.e., the property tables.

Useful Ideal Gas Relation: The Combined Gas Law

By writing the ideal gas equation twice for a fixed mass and simplifying, the properties of an ideal gas at two different states are related by

$$m_1 = m_2$$

or

$$\frac{P_1 V_1}{R T_1} = \frac{P_2 V_2}{R T_2}$$

But, the gas constant is _____ (fill in the blank), so

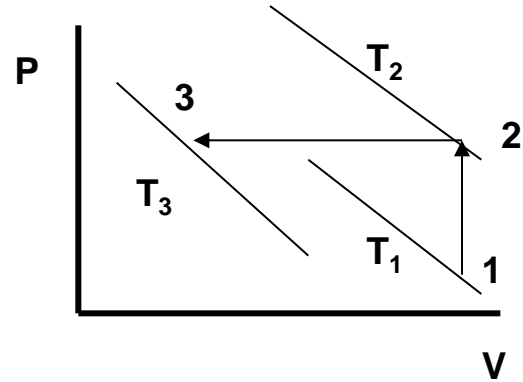
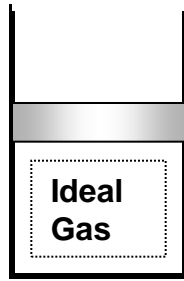
$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

Example 2-7

An ideal gas having an initial temperature of 25°C under goes the two processes described below. Determine the final temperature of the gas.

Process 1-2: The volume is held constant while the pressure doubles.

Process 2-3: The pressure is held constant while the volume is reduced to one-third of the original volume.



Process 1-3:

$$m_1 = m_3$$

or

$$\frac{P_1 V_1}{T_1} = \frac{P_3 V_3}{T_3}$$

but $V_3 = V_1/3$ and $P_3 = P_2 = 2P_1$

Therefore,

$$T_3 = T_1 \frac{P_3}{P_1} \frac{V_3}{V_1}$$

$$T_3 = T_1 \frac{2P_1}{P_1} \frac{V_1/3}{V_1} = \frac{2}{3} T_1$$

$$T_3 = \frac{2}{3} (25 + 273) K = 198.7 K = -74.3^\circ C$$

Other Equations of State

Many attempts have been made to keep the simplicity of the ideal gas equation of state but yet account for the intermolecular forces and volume occupied by the particles. Three of these are

van der Waals:

$$\left(P + \frac{a}{v^2}\right)(v - b) = RT$$

where

$$a = \frac{27R^2T_{cr}^2}{64P_{cr}} \quad \text{and} \quad b = \frac{RT_{cr}}{8P_{cr}}$$

Extra Assignment

When plotted on the P - v diagram, the critical isotherm has a point of inflection at the critical point. Use this information to verify the equations for van der Waals' constants a and b .

Beattie-Bridgeman:

$$P = \frac{R_u T}{\bar{v}^2} \left(1 - \frac{c}{\bar{v} T^3} \right) (\bar{v} + B) - \frac{A}{\bar{v}^2}$$

where

$$A = A_o \left(1 - \frac{a}{\bar{v}} \right) \quad \text{and} \quad B = B_o \left(1 - \frac{b}{\bar{v}} \right)$$

The constants a , b , c , A_o , B_o for various substances are found in Table 3-4.

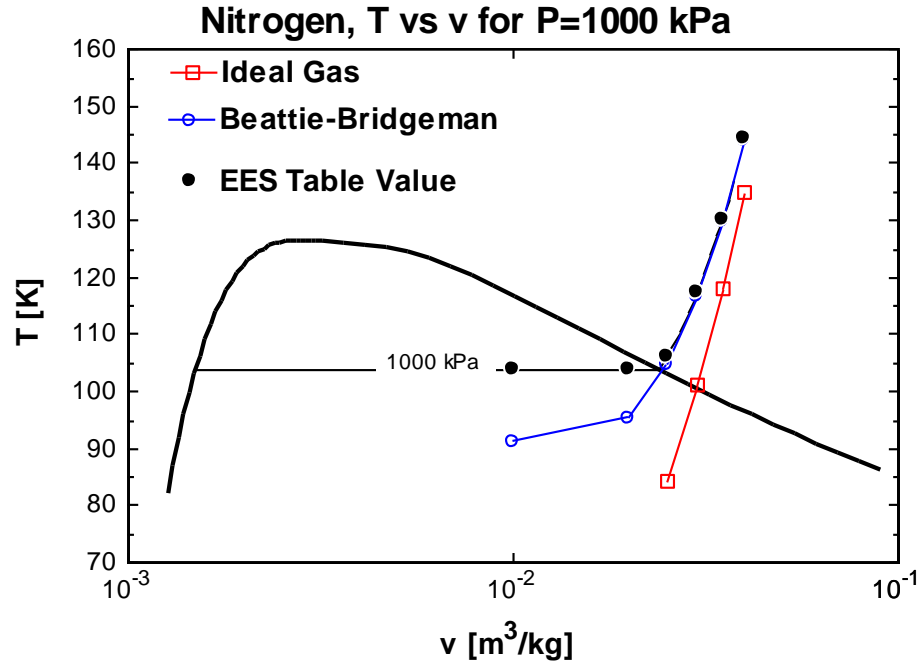
Benedict-Webb-Rubin:

$$P = \frac{R_u T}{\bar{v}} + \left(B_o R_u T - A_o - \frac{C_o}{T^2} \right) \frac{1}{\bar{v}^2} + \frac{b R_u T - a}{\bar{v}^3} \\ + \frac{a\alpha}{\bar{v}^6} + \frac{c}{\bar{v}^3 T^2} \left(1 + \frac{\gamma}{\bar{v}^2} \right) e^{-\gamma/\bar{v}^2}$$

The constants for various substances appearing in the Benedict-Webb-Rubin equation are given in Table 3-4.

Example 2-8

Compare the results from the ideal gas equation, the Beattie-Bridgeman equation, and the EES software for nitrogen at 1000 kPa. The following is an EES solution to that problem.



Notice that the results from the Beattie-Bridgeman equation compare well with the actual nitrogen data provided by EES in the gaseous or superheated region. However, neither the Beattie-Bridgeman equation nor the ideal gas equation provides adequate results in the two-phase region, where the gas (ideal or otherwise) assumption fails.